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HADRONIC MATHEMATICS, MECHANICS AND CHEMISTRY

Volume II:

Isodual Theory of Antimatter Antigravity and Spacetime Machines

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This volume is dedicated to the memory of

Professor Grigorios Tsagas

in recognition of his pioneering work on the Lie-Santilli isotheory.

Contents

Foreword

Mathematics is a subject which possibly finds itself in a unique position in academia in that it is viewed as both an Art and a Science. Indeed, in different universities, graduates in mathematics may receive Bachelor Degrees in Arts or Sciences. This probably reflects the dual nature of the subject. On the one hand, it may be studied as a subject in its own right. In this sense, its own beauty is there for all to behold; some as serene as da Vinci's "Madonna of the Rocks", other as powerful and majestic as Michelangelo's glorious ceiling of the Sistine Chapel, yet more bringing to mind the impressionist brilliance of Monet's Water Lily series. It is this latter example, with the impressionists interest in light, that links up with the alternative view of mathematics; that view which sees mathematics as the language of science, of physics in particular since physics is that area of science at the very hub of all scientific endeavour, all other branches being dependent on it to some degree. In this guise, however, mathematics is really a tool and any results obtained are of interest only if they relate to what is found in the real world; if results predict some effect, that prediction must be verified by observation and/or experiment. Again, it may be remembered that physics is really a collection of related theories. These theories are all manmade and, as such, are incomplete and imperfect. This is where the work of Ruggero Santilli enters the scientific arena.

Although "conventional wisdom" dictates otherwise, both the widely accepted theories of relativity and quantum mechanics, particularly quantum mechanics, are incomplete. The qualms surrounding both have been muted but possibly more has emerged concerning the inadequacies of quantum mechanics because of the people raising them. Notably, although it is not publicly stated too frequently, Einstein had grave doubts about various aspects of quantum mechanics. Much of the worry has revolved around the role of the observer and over the question of whether quantum mechanics is an objective theory or not. One notable contributor to the debate has been that eminent philosopher of science, Karl Popper. As discussed in my book, "Exploding a Myth", Popper preferred to refer to the experimentalist rather than observer, and expressed the view that that person played the same role in quantum mechanics as in classical mechanics. He felt, therefore, that such a person was there to test the theory. This is totally opposed to the Copenhagen Interpretation which claims that "objective reality has evaporated" and "quantum mechanics does not represent particles, but rather our knowledge, our observations, or our consciousness, of particles". Popper points out that, over the years, many eminent physicists have switched allegiance from the pro-Copenhagen view. In some ways, the most important of these people was David Bohm, a greatly respected thinker on scientific matters who wrote a book presenting the Copenhagen view of quantum mechanics in minute detail. However, later, apparently under Einstein's influence, he reached the conclusion that his previous view had been in error and also declared the total falsity of the constantly repeated dogma that the quantum theory is complete. It was, of course, this very question of whether or not quantum mechanics is complete which formed the basis of the disagreement between Einstein and Bohr; Einstein stating "No", Bohr "Yes".

However, where does Popper fit into anything to do with Hadronic Mechanics? Quite simply, it was Karl Popper who first drew public attention to the thoughts and ideas of Ruggero Santilli. Popper reflected on, amongst other things, Chadwick's neutron. He noted that it could be viewed, and indeed was interpreted originally, as being composed of a proton and an electron. However, again as he notes, orthodox quantum mechanics offered no viable explanation for such a structure. Hence, in time, it became accepted as a new particle. Popper then noted that, around his (Popper's) time of writing, Santilli had produced an article in which the "first structure model of the neutron" was revived by "resolving the technical difficulties which had led, historically, to the abandonment of the model". It is noted that Santilli felt the difficulties were all associated with the assumption that quantum mechanics applied within the neutron and disappeared when a generalised mechanics is used. Later, Popper goes on to claim Santilli to belong to a new generation of scientists which seemed to him to move on a different path. Popper identifies quite clearly how, in his approach, Santilli distinguishes the region of the arena of incontrovertible applicability of quantum mechanics from nuclear mechanics and hadronics. He notes also his most fascinating arguments in support of the view that quantum mechanics should not, without new tests, be regarded as valid in nuclear and hadronic mechanics.

Ruggero Santilli has devoted his life to examining the possibility of extending the theories of quantum mechanics and relativity so that the new more general theories will apply in situations previously excluded from them. To do this, he has had to go back to the very foundations and develop new mathematics and new mathematical techniques. Only after these new tools were developed was he able to realistically examine the physical situations which originally provoked this lifetime's work. The actual science is his, and his alone, but, as with the realization of all great endeavours, he has not been alone. The support and encouragement he has received from his wife Carla cannot be exaggerated. In truth, the scientific achievements of Ruggero Santiili may be seen, in one light, as the results of a team effort; a team composed of Ruggero himself and Carla Gandiglio in Santilli. The theoretical foundations of the entire work are contained in this volume; a volume which should be studied rigorously and with a truly open mind by the scientific community at large. This volume contains work which might be thought almost artistic in nature and is that part of the whole possessing the beauty so beloved of mathematicians and great artists. However, the scientific community should reserve its final judgement until it has had a chance to view the experimental and practical evidence which may be produced later in support of this elegant new theoretical framework.

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September 8, 2007

Preface

The author has indicated various times in his works that Albert Einstein has been the biggest scientist of the 20-th century, but also the most exploited scientist in history, because organized academic, financial and ethnic interests on Einstein have pushed the validity of his views way beyond the conditions of his original conception, by therefore turning what is supposed to be a serious scientific process into a pool of often ascientific conduits generally manipulated for personal gains.

This volume presents a solution of one of the several scientific imbalances of historical proportions caused by said ascientific interests in science, the abuse of academic authority and public funds to impose Einstein 's special and general relativity for the treatment of antimatter, while in the scientific reality Einsteinian theories have no means for a quantitative classical differentiation between neutral matter and antimatter, and even when assumed for charged classical particles, their operator image is a particle (rather than the correct charge conjugated antiparticle) with the wrong sign of the charge.

To defend the name of Albert Einstein, it must be noted that antimatter had yet to be discovered at the time of the formulation of his theories. Hence, the entire responsibility of this large scientific imbalance, and the expected severe judgment by posterity, must solely rest with said organized academic, financial and ethnic interests that extended for personal gains Einstein's views beyond the conditions of their original conception without a serious scrutiny.

The solution presented in this volume of the historical imbalance between matter and antimatter is based on the necessary development of a *new mathematics,* today called *Santilli isodual mathematics,* allowing for the first time the classical representation of antimatter as an anti-isomorphic image of that for matter. The isodual conjugation then persists under quantization, where it turns out to be equivalent to charge conjugation, thus restoring a full equivalence and scientific democracy in the treatment of matter and antimatter at all levels, from Newtonian mechanics to second quantization. The resulting *isodual theory of antimatter* then verifies, by conception and construction, all available experimental data on antimatter at the classical and operator level.

It should be stressed that, by no mans, the isodual theory of antimatter is presented as final, or complete or unique, because so many intriguing problems remain open. However, its dismissal in the absence of an alternative broadening of Einsteinian doctrines must be denounced as scientific corruption for personal gains because the appropriate broadening of Einsteinian doctrines for antimatter is indeed open to scientific debates, but not its need.

An illustration of the damage caused to human knowledge by said ascientific interests on Einstein is given by antigravity. Said interests have dismissed, disrupted and jeopardized for over half a century professional research on the possible antigravity between matter and antimatter (here referred to gravitational repulsion) on grounds that it is not predicted by Einstein's theories.

The need to contain said ascientific interests is rendered evident by the above indicated fact that Einstein's special and general relativities have no means for a quantitative classical differentiations between a neutral matter star and its antimatter counterpart. Under these conditions, the abuse of Einstein's name must be denounced as scientific corruption for personal gains by any person who cares about human dignity, let alone scientific knowledge.

A primary objective of this volume is to show that, once ascientific interests in science are cut out, and a theory for the proper classical and operator formulation of antimatter is worked out, *gravitational repulsion between matter and antimatter is mandated by all available theoretical and experimental evidence, with no credible objection on record.*

In this volume, we also review a proposed experiment to test the gravity of positrons in horizontal flight in a vacuum tube that has been qualified by independent experimentalists in the field as being readily feasible with current technologies and, above all, resolutory.

Yet, even the consideration of this so basic an experiment has been denied by SLAC, CERN, JINR and various other laboratories throughout the world because such a consideration would imply doubts on the universal validity of Einsteinian doctrines, by illustrating in this way the alarming dimension, diversification and capillary nature of ascientific interests at physics laboratories around the world.

The final objective of this volume is to illustrate that the expected experimental verification of antigravity between matter and antimatter will imply advances in human knowledge simply beyond our imagination at this time, such as a fully causal *spacetime geometric locomotion,* here referred to motion in space and time via the alteration of the local geometry, although not for ordinary matter or antimatter, but for a particular form of matter and antimatter called isoselfdual.

It is written in history that, following the achievement of control with protracted impunity, individuals lose the understanding of the self-damaging character of their actions. It is also written in history that people have the structure they want or deserve.

In view of the above ascientific condition of science, no basic advance on antimatter, antigravity and other far reaching frontiers is possible without the joint consideration of issues pertaining to scientific ethics and accountability particularly when ascientific interests in science are permitted to operate, by vile subservience or complicity, under public financial support.

To put it bluntly, the judgment expected by posterity on our contemporary scientific community will crucially depend on its capability to identify, denounce and contain ascientific and consequently asocial interests in science.

Ruggero Maria Santilli
January 19, 2008

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The underwriter Ruggero Maria Santilli states the following:

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This legal notice has been made necessary because, as shown in Section 1.5, the author has been dubbed "the most plagiarized scientist of the 20-th century," as it is the case of the thousands of papers in deformations published without any quotation of their origination by the author in 1967. These, and other attempted paternity frauds, have forced the author to initiate legal action reported in web site [1].

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In faith

Ruggero Maria Santilli

U. S. Citizen acting under the protection of the First Amendment of the U. S. Constitution guaranteeing freedom of expression particularly when used to contain asocial misconducts.

Tarpon Springs, Florida, U. S. A.

October 11, 2007

[1] International Committee on Scientific Ethics and Accountability
http://www.scientificethics.org

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Chapter 2

ISODUAL THEORY OF POINT-LIKE ANTIPARTICLES

2.1 ELEMENTS OF ISODUAL MATHEMATICS

2.1.1 Isodual Unit, Isodual Numbers and Isodual Fields

The first comprehensive study of the isodual theory for point-like antiparticles has been presented by the author in monograph [34]. However, the field is subjected to continuous developments following its first presentation in papers [1] of 1985. Hence, it is important to review the most recent formulation of the isodual mathematics in sufficient details to render this monograph self-sufficient.

In this section, we identify only those aspects of isodual mathematics that are essential for the understanding of the physical profiles presented in the subsequent sections of this chapter. We begin with a study of the most fundamental elements of all mathematical and physical formulations, units, numbers and fields, from which all remaining formulations can be uniquely and unambiguously derived via simple compatibility arguments. To avoid un-necessary repetitions, we assume the reader has a knowledge of the basic mathematics used for the classical and operator treatment of matter, including a knowledge of the fields of real, complex and quaternionic numbers. The symbol † usec in this chapter denotes conventional Hermitean conjugation, namely, transpose t plus complex conjugation ^{c}. Hence, for real numbers n we have $n^\dagger = n$, for complex numbers a we have $a^\dagger = a^c$ and for qauternions q we have $q^\dagger = q^{tc}$.

DEFINITION 2.1.1: Let $F = F(a, +, \times)$ be a field (of characteristic zero), namely a ring with elements given by real number $a = n,\ F = R(n, +, \times)$, complex numbers $A = c,\ F = C(c, +, \times)$, or quaternionic numbers $a = q,\ F = Q(q, +, \times)$, *with conventional sum* $a+b$ *verifying the commutative law*

$$a+b=b+a=c\in F, \tag{2.1.1}$$

the associative law

$$(a+b)+c=a+(b+c)=d\in F, \tag{2.1.2}$$

conventional product $a\times b$ *verifying the associative law*

$$(a\times b)\times c=a\times(b\times c)=e\in F, \tag{2.1.3}$$

(but not necessarily the commutative law, $a\times b\neq b\times a$ *since the latter is violated by quaternions), and the right and left distributive laws*

$$(a+b)\times c=a\times c+b\times c=f\in F, \tag{2.1.4a}$$

$$a\times(b+c)=a\times b+a\times c=g\in F, \tag{2.1.4b}$$

left and right additive unit 0,

$$a+0=0+a=a\in F, \tag{2.1.5}$$

and left and right multiplicative unit I,

$$a\times I=I\times a=a\in F, \tag{2.1.6}$$

$\forall a,\, b,\, c\in F$. *Santilli's isodual fields (first introduced in* Refs. [1] *and then presented in details in* Ref. [2]*) are rings* $F^d=F^d(a^d,+^d,\times^d)$ *with elements given by isodual numbers*

$$a^d=-a^\dagger,\ a^d\in F, \tag{2.1.7}$$

with associative and commutative isodual sum

$$a^d+^d b^d=-(a+b)^\dagger=c^d\in F^d, \tag{2.1.8}$$

associative and distributive isodual product

$$a^d\times^d b^d=a^d\times(I^d)^{-1}\times b^d=c^d\in F^d, \tag{2.1.9}$$

additive isodual unit $0^d=0$,

$$a^d+^d 0^d=0^d+^d a^d=a^d, \tag{2.1.10}$$

and multiplicative isodual unit $I^d=-I^\dagger$,

$$a^d\times^d I^d=I^d\times^d a^d=a^d,\forall a^d,b^d\in F^d. \tag{2.1.11}$$

The proof of the following property is elementary.

LEMMA 2.1.1 [1,2]: *Isodual fields are fields, namely, if F is a field, its image F^d under the isodual map is also a field.*

The above lemma establishes the property (first identified in Refs. [1]) that *the axioms of a field do not require that the multiplicative unit be necessarily positive-definite, because the same axioms are also verified by negative-definite units.* The proof of the following property is equally simple.

LEMMA 2.1.2 [1,2]: *Fields F and their isodual images F^d are anti-isomorphic to each other.*

Lemmas 2.1.1 and 1.2.2 illustrate the origin of the name "isodual mathematics". In fact, to represent antimatter the needed mathematics must be a suitable "dual" of conventional mathematics, while the prefix "iso" is used in its Greek meaning of preserving the original axioms.

It is evident that for real numbers we have

$$n^d = -n, \tag{2.1.12}$$

while for complex numbers we have

$$c^d = (n_1 + i \times n_2)^d = -n_1 + i \times n_2 = -\bar{c}, \tag{2.1.13}$$

with a similar formulation for quaternions.

It is also evident that, for consistency, *all operations on numbers must be subjected to isoduality when dealing with isodual numbers.* This implies: the *isodual powers*

$$(a^d)^{n^d} = a^d \times^d a^d \times^d a^d \dots \tag{2.1.14}$$

(n times, with n an integer); the *isodual square root*

$$a^{d(1/2)^d} = -\sqrt{-a^\dagger}^\dagger, a^{d(1/2)^d} \times^d a^{d(1/2)^d} = a^d, \quad 1^{d(1/2)^d} = -i; \tag{2.1.15}$$

the *isodual quotient*

$$a^d /^d b^d = -(a^\dagger / b^\dagger) = c^d, \; b^d \times^d c^d = a^d; \tag{2.1.16}$$

etc.

An important property for the characterization of antimatter is the following:

LEMMA 2.1.3. [2]: *isodual fields have a negative-definite norm, called isodual norm,*

$$|a^d|^d = |a^\dagger| \times I^d = -(aa^\dagger)^{1/2} < 0, \tag{2.1.17}$$

where $|\ldots|$ *denotes the conventional norm.*

For isodual real numbers we therefore have the isodual isonorm

$$|n^d|^d \;=\; -|n| < 0, \tag{2.1.18}$$

and for isodual complex numbers we have

$$|c^d|^d \;=\; -|\bar{c}| \;=\; -(c\bar{c})^{1/2} \;=\; -(n_1^2 + n_2^2)^{1/2}. \tag{2.1.19}$$

LEMMA 2.1.4 [2]: *All quantities that are positive-definite when referred to positive units and related fields of matter (such as mass, energy, angular momentum, density, temperature, time, etc.) become negative-definite when referred to isodual units and related isodual fields of antimatter.*

As recalled Chapter 1, antiparticles have been discovered in the *negative-energy solutions* of Dirac's equation and they were originally thought to evolve *backward in time* (Stueckelberg, Feynman, and others, see Refs. [1,2] of Chapter 1). The possibility of representing antiparticles via isodual methods is therefore visible already from these introductory notions.

The main novelty is that the conventional treatment of negative-definite energy and time was (and still is) referred to the conventional unit $+1$. This leads to a number of contradictions in the physical behavior of antiparticles.

By comparison, *negative-definite physical quantities of isodual theories are referred to a negative-definite unit* $I^d < 0$. This implies a mathematical and physical equivalence between *positive-definite quantities referred to positive-definite units, characterizing matter, and negative-definite quantities referred to negative-definite units, characterizing antimatter.* These foundations then permit a novel characterization of antimatter beginning at the *Newtonian* level, and then persisting at all subsequent levels.

DEFINITION 2.1.2 [2]: *A quantity is called isoselfdual when it coincides with its isodual.*

It is easy to verify that the imaginary unit is isoselfdual because

$$i^d \;=\; -i^\dagger \;=\; -\bar{i} \;=\; -(-i) \;=\; i. \tag{2.1.20}$$

This property permits a better understanding of the isoduality of complex numbers that can be written explicitly

$$c^d \;=\; (n_1 + i \times n_2)^d \;=\; n_1^d + i^d \times^d n_2^d \;=\; -n_1 + i \times n_2 \;=\; -\bar{c}. \tag{2.1.21}$$

The above property will be important to prove the equivalence of isoduality and charge conjugation at the operator level.

As we shall see, *isoselfduality is a new fundamental view of nature* with deep physical implications, not only in classical and quantum mechanics but also in cosmology. For instance we shall see that Dirac's gamma matrices are isoselfdual, thus implying a basically new interpretation of this equation that has remained unidentified for about one century. We shall also see that, when applied to cosmology, isoselfduality implies equal distribution of matter and antimatter in the universe, with identically null total physical characteristic, such as identically null total time, identically null total mass, etc.

We should also indicate that we have assumed the isoduality of the multiplication, $\times \rightarrow \times^d = \times(-1)\times = -\times$, but *not* that of the sum, $+ \rightarrow +^d = +(-1)+ = -$. This approach may not appear entirely motivated to the mathematically inclined reader because *fields are invariant under the above defined isoduality of the sum* due to the invariance of the additive unit, $0 \rightarrow 0^d \equiv 0$ (although fields are not invariant under the isoduality of the product due to the lack of invariance of the multiplicative unit, $1 \rightarrow 1^d = -1$).

The above decision is motivated by pragmatic, rather than mathematical arguments and, more specifically, for compatibility with the more general isofields and genofields, studied in the following chapters. In fact, at the latter broader levels, we have the loss of the invariance of the axioms of a field under these broader liftings of the sum. In turn, the loss of the field axioms cause the consequential inapplicability of the theory for physical applications as currently known, that is, based on "numbers" as rings verifying the axioms of a field, thus admitting a right and left, well defined, multiplicative unit representing the selected units of measurements.

It should also be stressed that, to avoid apparent inconsistencies, the isodual conjugation must be applied to all numbers and all their multiplications (or divisions). For instance, the isodual of a real numbers $n = n \times 1$ is given by $n^d \times^d 1^d = -n \times 1 = -n$ and not by $n^d \times 1^d = n$.

We assume the reader is aware of the emergence here of *new numbers,* those with a negative unit, that have no connection with ordinary negative numbers and are the true foundations of the isodual theory of antimatter.

2.1.2 Isodual Functional Analysis

All conventional and special functions and transforms, as well as functional analysis at large, must be subjected to isoduality for consistent applications, resulting in the simple, yet unique and significant *isodual functional analysis*, studied by Kadeisvili [3], Santilli [4] and others.

We here mention the *isodual trigonometric functions*

$$\sin^d \theta^d = -\sin(-\theta), \quad \cos^d \theta^d = -\cos(-\theta), \tag{2.1.22}$$

with related basic property

$$\cos^{d\,2d}\theta^d +^d \sin^{d\,2d}\theta^d \;=\; 1^d \;=\; -1, \tag{2.1.23}$$

the *isodual hyperbolic functions*

$$\sinh^d w^d \;=\; -\sinh(-w),\; \cosh^d w^d \;=\; -\cosh(-w), \tag{2.1.24}$$

with related basic property

$$\cosh^{d\,2d} w^d -^d \sinh^{d\,2d} w^d \;=\; 1^d \;=\; -1, \tag{2.1.25}$$

the *isodual logarithm* and the *isodual exponentiation* defined respectively by

$$\log^d n^d \;=\; -\log(-n), \tag{2.1.26a}$$

$$e_d^{X^d} = 1^d + X^d/^d 1!^d + X^{d2d}/^d 2!^d + \ldots = -e^X, \tag{2.1.26b}$$

etc. Interested readers can then easily construct the isodual image of special functions, transforms, distributions, etc.

2.1.3 Isodual Differential and Integral Calculus

Contrary to a rather popular belief, the differential calculus is indeed dependent on the assumed unit. This property is not so transparent in the conventional formulation because the basic unit is the trivial number +1. However, the dependence of the unit emerges rather forcefully under its generalization.

The *isodual differential calculus*, first introduced by Santilli in Ref. [5a], is characterized by the *isodual differentials*

$$d^d x^k \;=\; I^d \times dx^k \;=\; -dx^k,\;\; d^d x_k \;=\; -dx_k, \tag{2.1.27}$$

with corresponding *isodual derivatives*

$$\partial^d/^d\partial^d x^k \;=\; -\partial/\partial x^k,\;\; \partial^d/^d\partial^d x_k \;=\; -\partial/\partial x_k, \tag{2.1.28}$$

and related isodual properties.

Note that *conventional differentials are isoselfdual*, i.e.,

$$(dx^k)^d \;=\; d^d x^{kd} \equiv d\,x^k, \tag{2.1.29}$$

but *derivatives are not isoselfdual*,

$$[\partial f/\partial x^k]^d \;=\; -\partial^d f^d/^d\partial^d x^{kd}. \tag{2.1.30}$$

The above properties explain why the isodual differential calculus remained undiscovered for centuries.

Other notions, such as the *isodual integral calculus,* can be easily derived and shall be assumed as known hereon.

2.1.4 Lie-Santilli Isodual Theory

Let $\mathbf{L}$ be an n-dimensional Lie algebra in its regular representation with universal enveloping associative algebra $\xi(\mathbf{L})$, $[\xi(\mathbf{L})]^- \approx \mathbf{L}$, n-dimensional unit $I = Diag.(1, 1, \ldots, 1)$, ordered set of Hermitian generators $X = X^\dagger = \{X_k\}$, $k = 1, 2, \ldots, n$, conventional associative product $X_i \times X_j$, and familiar Lie's Theorems over a field $F(a, +, \times)$.

The *Lie-Santilli isodual theory* was first submitted in Ref. [1] and then studied in Refs. [4-7] as well as by other authors [23-31]. *The isodual universal associative algebra* $[\xi(\mathbf{L})]^d$ is characterized by the *isodual unit* I^d, *isodual generators* $X^d = -X$, and isodual associative product

$$X_i^d \times^d X_j^d = -X_i \times X_j, \tag{2.1.31}$$

with corresponding infinite-dimensional basis characterized by the *Poincaré-Birkhoff-Witt-Santilli isodual theorem*

$$I^d, X_i^d \times^d X_j^d, \ \ i \leq j; \ \ X_i^d \times^d X_j^d \times X_k^d, \ i \leq j \leq k, \ldots \tag{2.1.32}$$

and related *isodual exponentiation* of a generic quantity A^d

$$e^{dA^d} = I^d + A^d/^d 1!^d + A^d \times^d A^d/^d 2!^d + \ldots = -e^{A^\dagger}, \tag{2.1.33}$$

where e is the conventional exponentiation.

The attached *Lie-Santilli isodual algebra* $\mathbf{L}^d \approx (\xi^d)^-$ over the isodual field $F^d(a^d, +^d, \times^d)$ is characterized by the *isodual commutators* [1]

$$[X_i^d, ^d X_j^d] = -[X_i, X_j] = C_{ij}^{k^d} \times^d X_k^d. \tag{2.1.34}$$

with classical realizations given in Section 2.2.6.

Let G be a conventional, connected, n-dimensional Lie transformation group on a metric (or pseudo-metric) space $S(x, g, F)$ admitting $\mathbf{L}$ as the Lie algebra in the neighborhood of the identity, with generators X_k and parameters $w = \{w_k\}$.

The *Lie-Santilli isodual transformation group* G^d admitting the isodual Lie algebra $\mathbf{L}^d$ in the neighborhood of the isodual identity I^d is the n-dimensional group with generators $X^d = \{-X_k\}$ and parameters $w^d = \{-w_k\}$ over the isodual field F^d with generic element [1]

$$U^d(w^d) = e^{d^{i^d \times^d w^d \times^d X^d}} = -e^{i \times (-w) \times X} = -U(-w). \tag{2.1.35}$$

The *isodual symmetries* are then defined accordingly via the use of the isodual groups G^d and they are anti-isomorphic to the corresponding conventional symmetries, as desired. For additional details, one may consult Ref. [4,5b].

In this chapter we shall therefore use the *conventional Poincaré, internal and other symmetries* for the characterization of *matter*, and the *Poincaré-Santilli, internal and other isodual symmetries* for the characterization of *antimatter*.

2.1.5 Isodual Euclidean Geometry

Conventional (vector and) metric spaces are defined over conventional fields. It is evident that the isoduality of fields requires, for consistency, a corresponding isoduality of (vector and) metric spaces. The need for the isodualities of all quantities acting on a metric space (e.g., conventional and special functions and transforms, differential calculus, etc.) becomes then evident.

DEFINITION 2.1.3: Let $S = S(x, g, R)$ be a conventional N-dimensional metric or pseudo-metric space with local coordinates $x = \{x^k\}$, $k = 1, 2, \ldots, N$, nowhere degenerate, sufficiently smooth, real-valued and symmetric metric $g(x, \ldots)$ and related invariant

$$x^2 \;=\; (x^i \times g_{ij} \times x^j) \times I, \tag{2.1.36}$$

over the reals R. The isodual spaces, first introduced in Ref. [1] (see also Refs. [4,5] and, for a more recent account, Ref. [22]), *are the spaces $S^d(x^d,\, g^d,\, R^d)$ with isodual coordinates $x^d = x^d = -x^t$ (where t stands for transposed), isodual metric*

$$g^d(x^d, \ldots) \;=\; -g^\dagger(-x^\dagger, \ldots) \;=\; -g(-x^t, \ldots), \tag{2.1.37}$$

and isodual interval

$$\begin{aligned}(x-y)^{d2\,d} \;&=\; [(x-y)^{id} \times^d g^d_{ij} \times^d (x-y)^{jd}] \times I^d = \\ &=\; [(x-y)^i \times g^d_{ij} \times (x-y)^j] \times I^d,\end{aligned} \tag{2.1.38}$$

defined over the isodual field $R^d \;=\; R^d(n^d, +^d, \times^d)$ with the same isodual isounit I^d.

The basic nonrelativistic space of our analysis is the three-dimensional *isodual Euclidean space* [1,9],

$$E^d(r^d, \delta^d, R^d) : r^d \;=\; \{r^{kd}\} \;=\; \{-r^k\} \;=\; \{-x, -y, -z\}, \tag{2.1.39a}$$

$$\begin{aligned}\delta^d \;&=\; -\delta \;=\; Diag.(-1, -1, -1),\\ I^d \;&=\; -I \;=\; Diag.(-1, -1, -1).\end{aligned} \tag{2.1.39b}$$

The *isodual Euclidean geometry* is the geometry of the isodual space E^d over R^d and it is given by a step-by-step isoduality of all the various aspects of the conventional geometry (see monograph [5a] for details).

By recalling that the norm on R^d is negative-definite, the *isodual distance* among two points on an isodual line is also negative definite and it is given by

$$D^d \;=\; D \times I^d \;=\; -D, \tag{2.1.40}$$

where D is the conventional distance. Similar isodualities apply to all remaining notions, including the notions of parallel and intersecting isodual lines, the Euclidean axioms, etc.

The *isodual sphere* with radius $R^d = -R$ is the perfect sphere on E^d over R^d and, as such, it has *negative radius* (Figure 2.1),

$$R^{d2d} = (x^{d2d} + y^{d2d} + z^{d2d}) \times I^d = \\ = (x^2 + y^2 + z^2) \times I = R^2. \quad (2.1.41)$$

Note that the above expression coincides with that for the conventional sphere. This illustrates the reasons, following about one century of studies, the isodual rotational group and symmetry where identified for the first time in Ref. [1]. Note, however, that the latter result required the prior discovery of *new numbers,* those with a negative unit.

A similar characterization holds for other isodual shapes characterizing antimatter in our isodual theory.

LEMMA 2.1.5: The isodual Euclidean geometry on E^d over R^d is anti-isomorphic to the conventional geometry on E over R.

The group of isometries of E^d over R^d is the *isodual Euclidean group* $E^d(3) = \mathcal{R}^d(\theta^d) \times^d T^d(3)$ where $\mathcal{R}^d(\theta)$ is the isodual group of rotations first introduced in Ref. [1], and $T^d(3)$ is the isodual group of translations (see also Ref. [5a] for details).

2.1.6 Isodual Minkowskian Geometry

Let $M(x, \eta, R)$ be the conventional Minkowski spacetime with local coordinates $x = (r^k,\, t) = (x^\mu),\ k = 1,\, 2,\, 3,\ \mu = 1,\, 2,\, 3,\, 4$, metric $\eta = Diag.(1,\, 1,\, 1, -1)$ and basic unit $I = Diag.(1,\, 1,\, 1,\, 1)$ on the reals $R = R(n,\, +,\, \times)$.

The *Minkowski-Santilli isodual spacetime*, first introduced in Ref. [7] and studied in details in Ref. [8], is given by

$$M^d(x^d, \eta^d, R^d) : x^d = \{x^{\mu d}\} = \{x^\mu \times I^d\} = \{-r, -c_o t\} \times I, \quad (2.1.42)$$

with isodual metric and isodual unit

$$\eta^d = -\eta = Diag.(-1, -1, -1, +1), \quad (2.1.43a)$$

$$I^d = Diag.(-1, -1, -1, -1). \quad (2.1.43b)$$

The *Minkowski-Santilli isodual geometry* [8] is the geometry of isodual spaces M^d over R^d. The new geometry is also characterized by a simple isoduality of the conventional Minkowskian geometry as studied in details in memoir.

The fundamental symmetry of this chapter is given by the group of isometries of M^d over R^d, namely, the *Poincaré-Santilli isodual symmetry* [7,8]

$$P^d(3.1) = \mathcal{L}^d(3.1) \times T^d(3.1), \quad (2.1.44)$$

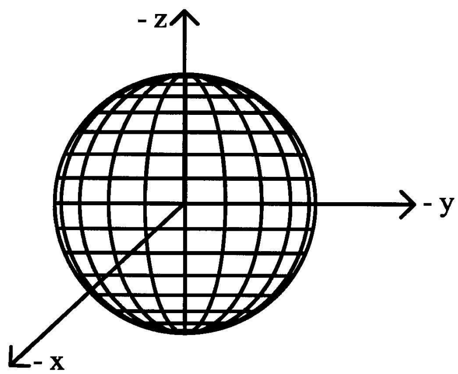

Figure 2.1. A schematic view of the isodual sphere on isodual Euclidean spaces over isodual fields. The understanding of the content of this chapter requires the knowledge that the isodual sphere and the conventional sphere coincide when inspected by an observer either in the Euclidean or in the isodual Euclidean space, due to the identity of the related expressions (2.1.36) and (2.1.38). This identity is at the foundation of the perception that antiparticles "appear" to exist in our space, while in reality they belong to a structurally different space coexisting within our own, thus setting the foundations of a "multidimensional universe" coexisting in the same space of our sensory perception. The reader should keep in mind that the isodual sphere is the idealization of the shape of an antiparticle used in this monograph.

where $\mathcal{L}^d(3.1)$ is the Lorentz-Santilli isodual group and $T^d(3.1)$ is the isodual group of translations.

2.1.7 Isodual Riemannian Geometry

Consider a Riemannian space $\Re(x, g, R)$ in $(3+1)$ dimensions [32] with basic unit $I = Diag.(1, 1, 1, 1)$, nowhere singular and symmetric metric $g(x)$ and related Riemannian geometry in local formulation (see, e.g., Ref. [27]).

The *Riemannian-Santilli isodual spaces* (first introduced in Ref. [11]) are given by

$$\begin{aligned}\Re^d(x^d, g^d, R^d): \ \ x^d &= \{-x^\mu\},\\ g^d &= -g(x), \ g \in \Re(x, g, R),\\ I^d &= Diag.(-1, -1, -1, -1)\end{aligned} \tag{2.1.45}$$

with interval

$$\begin{aligned}x^{2d} &= [x^{dt} \times^d g^d(x^d) \times^d x^d] \times I^d =\\ &= [x^t \times g^d(x^d) \times x] \times I^d \in R^d,\end{aligned} \tag{2.1.46}$$

where t stands for transposed.

The *Riemannian-Santilli isodual geometry* [8] is the geometry of spaces $\Re^d$ over R^d, and it is also given by step-by-step isodualities of the conventional geometry, including, most importantly, the isoduality of the differential and exterior calculus.

As an example, an *isodual vector field* $X^d(x^d)$ on $\Re^d$ is given by $X^d(x^d) = -X^t(-x^t)$. The *isodual exterior differential* of $X^d(x^d)$ is given by

$$D^d X^{kd}(x^d) \;=\; d^d X^{kd}(x^d) + \Gamma^{dk}_{i\,j} \times^d X^{id} \times^d d^d x^{jd} \;=\; DX^k(-x), \qquad (2.1.47)$$

where the Γ^d's are the components of the *isodual connection*. The *isodual covariant derivative* is then given by

$$X^{id}(x^d)_{|^{dk}} \;=\; \partial^d X^{id}(x^d)/^d \partial^d x^{kd} + \Gamma^{di}_{jk} \times^d X^{jd}(x^d) \;=\; -X^i(-x)_{|^k}. \qquad (2.1.48)$$

The interested reader can then easily derive the isoduality of the remaining notions of the conventional geometry.

It is an instructive exercise for the interested reader to work out in detail the proof of the following:

LEMMA 2.1.6 [8]: *The isodual image of a Riemannian space* $\Re^d(x^d, g^d, R^d)$ *is characterized by the following maps*:

$$Basic \quad Unit$$

$$I \;\rightarrow\; I^d \;=\; -I,$$

$$Metric$$

$$g \;\rightarrow\; g^d \;=\; -g, \qquad (2.1.49a)$$

$$Connection \quad Coefficients$$

$$\Gamma_{klh} \;\rightarrow\; \Gamma^d_{klh} \;=\; -\Gamma_{klh}, \qquad (2.1.49b)$$

$$Curvature \quad Tensor$$

$$R_{lijk} \;\rightarrow\; R^d_{lijk} \;=\; -R_{lijk}, \qquad (2.1.49c)$$

$$Ricci \quad Tensor$$

$$R_{\mu\nu} \;\rightarrow R^d_{\mu\nu} \;=\; -R_{\mu\nu}, \qquad (2.1.49d)$$

$$Ricci \quad Scalar$$

$$R \;\rightarrow\; R^d \;=\; R, \qquad (2.1.49e)$$

$$Einstein - Hilbert \quad Tensor$$

$$G_{\mu\nu} \;\rightarrow\; G^d_{\mu\nu} \;=\; -G_{\mu\nu}, \qquad (2.1.49f)$$

$$Electromagnetic \quad Potentials$$

$$A_\mu \ \rightarrow \ A^d_\mu \ = \ -A_\mu, \tag{2.1.49g}$$

$$Electromagnetic \quad Field$$

$$F_{\mu\nu} \ \rightarrow \ F^d_{\mu\nu} \ = \ -F_{\mu\nu}, \tag{2.1.49h}$$

$$ElmEnergy - Momentum \quad Tensor$$

$$T_{\mu\nu} \ \rightarrow \ T^d_{\mu\nu} \ = \ -T_{\mu\nu}, \tag{2.1.49i}$$

In summary, the geometries significant for this study are: the *conventional Euclidean, Minkowskian and Riemannian geometries* used for the characterization of *matter*; and the *isodual Euclidean, Minkowskian and Riemannian geometries* used for the characterization of *antimatter*.

The reader can now begin to see the achievement of axiomatic compatibility between gravitation and electroweak interactions that is permitted by the isodual theory of antimatter. In fact, the latter is treated via negative-definite energy-momentum tensors, thus being compatible with the negative-energy solutions of electroweak interactions, therefore setting correct axiomatic foundations for a true grand unification studied in the next chapter.

2.2 CLASSICAL ISODUAL THEORY OF POINT-LIKE ANTIPARTICLES

2.2.1 Basic Assumptions

Thanks to the preceding study of isodual mathematics, we are now sufficiently equipped to resolve the scientific impasse caused by the absence of a classical theory of antimatter studied in Section 1.1.

As it is well known, the contemporary treatment of matter is characterized by *conventional mathematics*, here referred to ordinary numbers, fields, spaces, etc. with *positive units and norms*, thus having positive characteristics of mass, energy, time, etc.

In this chapter we study the *characterization of antimatter via isodual numbers, fields, spaces, etc., thus having negative-definite units and norms.* In particular, all characteristics of matter (and not only charge) change sign for antimatter when represented via isoduality.

The above characterization of antimatter evidently provides the correct conjugation of the charge at the desired classical level. However, by no means, the sole change of the sign of the charge is sufficient to ensure a consistent classical representation of antimatter. To achieve consistency, the theory must resolve the main problematic aspect of current classical treatments, the fact that their operator image is not the correct charge conjugate state (Section 2.1).

The above problematic aspect is indeed resolved by the isodual theory. The main reason is that, jointly with the conjugation of the charge, isoduality also conjugates *all* other physical characteristics of matter. This implies *two* channels of quantization, the conventional one for matter and a new *isodual quantization* for antimatter (see Section 2.3) in such a way that its operator image is indeed the charge conjugate of that of matter.

In this section, we study the physical consistency of the theory in its classical formulation. The novel isodual quantization, the equivalence of isoduality and charge conjugation and related operator issues are studied in the next section.

Beginning our analysis, we note that the isodual theory of antimatter resolves the traditional obstacles against negative energies and masses. In fact, *particles with negative energies and masses measured with negative units are fully equivalent to particles with positive energies and masses measured with positive units.* This result has permitted the elimination of sole use of second quantization for the characterization of antiparticles because antimatter becomes treatable at *all* levels, including second quantization.

The isodual theory of antimatter also resolves the additional, well known, problematic aspects of motion backward in time. In fact, *time moving backward measured with a negative unit is fully equivalent on grounds of causality to time moving forward measured with a positive unit.*

This confirms the plausibility of the first conception of antiparticles by Stueckelberg and others as moving backward in time (see the historical analysis in Ref. [1] of Chapter 1), and creates new possibilities for the ongoing research on the so-called "spacetime machine" studied in Chapter 5.

In this section, we construct the classical isodual theory of antimatter at the Newtonian, Lagrangian, Hamiltonian, Galilean, relativistic and gravitational levels; we prove its axiomatic consistency; and we verify its compatibility with available classical experimental evidence (that dealing with electromagnetic interactions only). Operator formulations and their experimental verifications will be studied in the next section.

2.2.2 Need for Isoduality to Represent All Time Directions

It is popularly believed that time has only two directions, the celebrated *Eddington's time arrows.* In reality, *time has four different directions* depending on whether motion is forward or backward and occurs in the future or in the past, as illustrated in Figure 2.2. In turn, the correct use of all four different directions of time is mandatory, for instance, in serious studies of bifurcations, as we shall see.

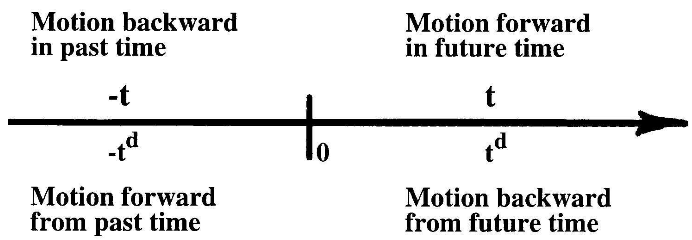

Figure 2.2. A schematic view of the "four different directions of time", depending on whether motion is forward or backward and occurs in the future or in the past. Due to the sole existence of one time conjugation, time reversal, the theoretical physics of the 20-th century missed two of the four directions of time, resulting in fundamental insufficiencies ranging from the lack of a deeper understanding of antiparticles to basic insufficiencies in biological structures and excessively insufficient cosmological views. It is evident that isoduality can indeed represent the two missing time arrows and this illustrates a basic need for the isodual theory.

It is evident that theoretical physics of the 20-th century could not explain all four directions of time, since it possessed only one conjugation, time reversal, and this explains the reason the two remaining directions of time were ignored.

It is equally evident that isoduality does indeed permit the representation of the two missing directions of time, thus illustrating its need.

We assume the reader is now familiar with the differences between time reversal and isoduality. Time reversal changes the direction of time while keeping the underlying space and units unchanged, while isoduality changes the direction of time while mapping the underlying space and units into different forms.

Unless otherwise specified, through the rest of this volume time t will be indicate *motion forward in future times,* $-t$ will indicate *motion backward in past times,* t^d will indicate motion backward from future times, and $-t^d$ will indicate *motion forward from past times.*

2.2.3 Experimental Verification of the Isodual Theory of Antimatter in Classical Physics

The experimental verification of the isodual theory of antimatter at the *classical* level is provided by the compliance of the theory with the only available experimental data, those on Coulomb interactions.

For that purpose, let us consider the Coulomb interactions under the customary notation that *positive (negative) forces represent repulsion (attraction)* when formulated in conventional Euclidean space.

Under such an assumption, the *repulsive* Coulomb force among two *particles* of negative charges $-q_1$ and $-q_2$ in Euclidean space $E(r,\,\delta,\,R)$ is given by

$$F \;=\; K\times(-q_1)\times(-q_2)/r\times r>0, \tag{2.2.1}$$

where K is a positive constant whose explicit value (here irrelevant) depends on the selected units, the operations of multiplication $\times$ and division $/$ are the conventional ones of the underlying field $R(n,\,+,\,\times)$.

Under isoduality to $E^d(r^d,\,\delta^d,\,R^d)$ the above law is mapped into the form

$$F^d \;=\; K^d\times^d(-q_1)^d\times^d(-q_2)^d/^d r^d\times^d r^d \;=\; -F<0, \tag{2.2.2}$$

where $\times^d=-\times$ and $/^d=-/$ are the isodual operations of the underlying field $R^d(n^d,\,+,\,\times^d)$.

But the isodual force $F^d=-F$ occurs in the isodual Euclidean space and it is, therefore, defined with respect to the unit -1. This implies that the reversal of the sign of a repulsive force measured with a negative unit also describes repulsion. As a result, isoduality correctly represents the *repulsive* character of the Coulomb force for two *antiparticles* with *positive* charges, a result first achieved in Ref. [9].

The formulation of the cases of two particles with positive charges and their antiparticles with negative charges is left to the interested reader.

The Coulomb force between a *particle* and an *antiparticle* can only be computed by *projecting the antiparticle in the conventional space of the particle or vice-versa*. In the former case we have

$$F \;=\; K\times(-q_1)\times(-q_2)^d/r\times r<0, \tag{2.2.3}$$

thus yielding an *attractive* force, as experimentally established. In the projection of the particle in the isodual space of the antiparticle, we have

$$F^d \;=\; K^d\times^d(-q_1)\times^d(-q_2)^d/^d r^d\times^d r^d>0. \tag{2.2.4}$$

But this force is now measured with the unit -1, thus resulting in being again *attractive*.

The study of Coulomb interactions of magnetic poles and other classical experimental data is left to the interested reader.

In conclusion, the isodual theory of antimatter correctly represents all available classical experimental evidence in the field.

2.2.4 Isodual Newtonian Mechanics

A central objective of this section is to show that the isodual theory of antimatter resolves the scientific imbalance of the 20-th century between matter and antimatter, by permitting the study of antimatter at *all* levels as occurring for matter. Such an objective can only be achieved by first establishing the existence of a

Newtonian representation of antimatter subsequently proved to be compatible with known operator formulations.

As it is well known, the Newtonian treatment of N *point-like particles* is based on a $7N$-dimensional representation space given by the Kronecker products of the Euclidean spaces of time t, coordinates r and velocities v (for the conventional case see Refs. [33,34]),

$$S(t,\, r,\, v) = E(t,\, R_t) \times E(r,\, \delta,\, R_r) \times E(v,\, \delta,\, R_v), \tag{2.2.5}$$

where

$$r = (r^k_a) = (r^1_a,\, r^2_a,\, r^3_a) = (x_a,\, y_a,\, z_a), \tag{2.2.6a}$$

$$v = (v_{ka}) = (v_{1a},\, v_{2a},\, v_{3a}) = (v_{xa},\, v_{ua},\, v_{za}) = dr/dt, \tag{2.2.6b}$$

$$\delta = Diag.(1,\, 1,\, 1),\; k = 1,\, 2,\, 3,\; a = 1,\, 2,\, 3, \ldots,\, N, \tag{2.2.6c}$$

and the base fields are trivially identical, i.e., $R_t = R_r = R_v$, since all units are assumed to have the trivial value $+1$, resulting in the trivial total unit

$$I_{tot} = I_t \times I_r \times I_v = 1 \times 1 \times 1 = 1. \tag{2.2.7}$$

The resulting basic equations are then given by the celebrated *Newton's equations for point-like particles*

$$m_a \times dv_{ka}/dt = F_{ka}(t,\, r,\, v),\; k = 1,\, 2,\, 3,\; a = 1,\, 2,\, 3, \ldots,\, N. \tag{2.2.8}$$

The basic space for the treatment of n antiparticles is given by the $7N$-dimensional *isodual space* [9]

$$S^d(t^d, r^d, v^d) \;=\; E^d(t^d, R^d_t) \times E^d(r^d, \delta^d, R^d) \times E^d(v^d, \delta^d, R^d), \tag{2.2.9}$$

with *isodual unit* and *isodual metric*

$$I^d_{Tot} \;=\; I^d_t \times I^d_r \times I^d_v, \tag{2.2.10a}$$

$$I^d_t \;=\; -1,\; I^d_r \;=\; I^d_v \;= Diag.(-1, -1, -1), \tag{2.2.10b}$$

$$\delta^d = Diag.(1^d, 1^d, 1^d) = Diag.(-1, -1, -1). \tag{2.2.10c}$$

We reach in this way the basic equations of this chapter, today known as the *Newton-Santilli isodual equations for point-like antiparticles,* first introduced in Ref. [4],[1]

$$m^d_a \times^d d^d v^d_{ka} /^d d^d t^d \;=\; F^d_{ka}(t^d, r^d, v^d), \tag{2.2.11}$$

$$k = x,\, y,\, z,\; a = 1,\, 2, \ldots,\, n,$$

[1]Note as necessary pre-requisites of the new Newton's equations, the prior discovery of isodual numbers, spaces and differential calculus.

whose experimental verification has been provided in the preceding section.

It is easy to see that the isodual formulation is anti-isomorphic to the conventional version, as desired, to such an extent that the two formulations actually coincide at the abstract, realization-free level.

Despite this axiomatic simplicity, the physical implications of the isodual theory of antimatter are rather deep. To begin their understanding, note that throughout the 20-th century it was believed that matter and antimatter exist in the same spacetime. In fact, as recalled earlier, charge conjugation is a map of our physical spacetime into itself.

One of the first physical implications of the Newton-Santilli isodual equations is that *antimatter exists in a spacetime co-existing, yet different than our own.* In fact, the isodual Euclidean space $E^d(r^d, \delta^d, R^d)$ co-exists within, but it is physically distinct from our own Euclidean space $E(r, \delta, R)$, and the same occurs for the full representation spaces $S^d(t^d, r^d, v^d)$ and $S(t, r, v)$.

The next physical implication of the Newton-Santilli isodual equations is the confirmation that *antimatter moves backward in time in a way as causal as the motion of matter forward in time* (again, because negative time is measured with a negative unit). In fact, the *isodual time* t^d is necessarily negative whenever t is our ordinary time. Alternatively, we can say that *the Newton-Santilli isodual equations provide the only known causal description of particles moving backward in time.*

Yet another physical implication is that *antimatter is characterized by negative mass, negative energy and negative magnitudes of other physical quantities.* As we shall see, these properties have the important consequence of eliminating the necessary use of Dirac's "hole theory."

The rest of this chapter is dedicated to showing that the above novel features are necessary in order to achieve a consistent representation of antimatter at all levels of study, from Newton to second quantization.

As we shall see, the physical implications are truly at the edge of imagination, such as: the existence of antimatter in a new spacetime different from our own constitutes the first known evidence of multi-dimensional character of our universe despite our sensory perception to the contrary; the achievement of a fully equivalent treatment of matter and antimatter implies the necessary existence of antigravity for antimatter in the field of matter (and vice-versa); the motion backward in time implies the existence of a causal spacetime machine (although restricted for technical reasons only to isoselfdual states); and other far reaching advances.

2.2.5 Isodual Lagrangian Mechanics

The second level of treatment of *matter* is that via the conventional *classical Lagrangian mechanics.* It is, therefore, essential to identify the corresponding formulation for *antimatter,* a task first studied in Ref. [4] (see also Ref. [9]).

A conventional (first-order) Lagrangian $L(t,\, r,\, v) = \frac{1}{2} \times m \times v^k \times v_k + V(t,\, r,\, v)$ on configuration space (2.2.5) is mapped under isoduality into the *isodual Lagrangian*

$$L^d(t^d,\, r^d,\, v^d) = -L(-t,\, -r,\, -v), \tag{2.2.12}$$

defined on isodual space (2.2.9).

In this way we reach the basic analytic equations of this chapter, today known as *Lagrange-Santilli isodual equations,* first introduced in Ref. [4]

$$\frac{d^d}{d^d t^d} d \frac{\partial^d L^d(t^d, r^d, v^d)}{\partial^d v^{kd}} d - \frac{\partial^d L^d(t^d, r^d, v^d)}{\partial^d r^{kd}} d = 0, \tag{2.2.13}$$

All various aspects of the *isodual Lagrangian mechanics* can then be readily derived.

It is easy to see that isodual equations (2.3.13) provide a *direct analytic representation* (i.e., a representation without integrating factors or coordinate transforms) of the isodual equations (2.2.11),

$$\frac{d^d}{d^d t^d} d \frac{\partial^d L^d(t^d, r^d, v^d)}{\partial^d v^{kd}} d - \frac{\partial^d L^d(t^d, r^d, v^d)}{\partial^d x^{kd}} d =$$

$$= m_k^d \times^d d^d v_k^d /^d d^d t^d - F_k^{d\,SA}(t,\, r,\, v) \;=\; 0. \tag{2.2.14}$$

The compatibility of the isodual Lagrangian mechanics with the primitive Newtonian treatment then follows.

2.2.6 Isodual Hamiltonian Mechanics

The *isodual Hamiltonian* is evidently given by [4,9]

$$H^d \;=\; p_k^d \times^d p^{dk} /^d 2^d \times^d m^d + V^d(t^d, r^d, v^d) \;=\; -H. \tag{2.2.15}$$

It can be derived from (nondegenerate) isodual Lagrangians via a simple isoduality of the Legendre transforms and it is defined on the $7N$-dimensional *isodual phase space (isocotangent bundle)*

$$S^d(t^d, r^d, p^d) \;=\; E^d(t^d, R_t^d) \times E^d(r^d, \delta^d, R^d) \times E^d(p^d, \delta^d, R^d). \tag{2.2.16}$$

The *isodual canonical action* is given by [4,9]

$$A^{\circ d} \;=\; \int_{t_1}^{t_2} (p_k^d \times^d d^d r^{kd} - H^d \times^d d^d t^d) \;=$$

$$= \int_{t_1}^{t_2} [R_\mu^{\circ d}(b^d) \times^d d^d b^{\mu d} - H^d \times^d d^d t^d], \tag{2.2.17a}$$

$$R^\circ \ = \ \{p, 0\}, \ b = \{x, p\}, \ \mu = 1, 2, \ldots, 6. \tag{2.2.17b}$$

Conventional variational techniques under simple isoduality then yield the fundamental canonical equations of this chapter, today known as *Hamilton-Santilli isodual equations* [4,24-31] that can be written in the disjoint r and p notation

$$\frac{d^d x^{kd}}{d^d t^d} = \frac{\partial^d H^d(t^d, x^d, p^d)}{\partial^d p_k^d}, \ \frac{d^d p_k^d}{d^d t^d} = -\frac{\partial^d H^d(t^d, x^d, p^d)}{\partial^d x^{dk}}, \tag{2.2.18}$$

or in the unified notation

$$\omega_{\mu\nu}^d \times^d \frac{d^d b^{d\nu}}{d^d t^d} \ = \ \frac{\partial^d H^d(t^d, b^d)}{\partial^d b^{d\mu}}, \tag{2.2.19}$$

where $\omega_{\mu\nu}^d$ is the *isodual canonical symplectic tensor*

$$(\omega_{\mu\nu}^d) \ = \ (\partial^d R_\nu^{\circ d} /^d \partial^d b^{d\mu} - \partial^d R_\mu^{\circ d} /^d \partial^d b^{d\nu}) = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} \ = \ (\omega^{\mu\nu}). \tag{2.2.20}$$

Note that isoduality maps the canonical symplectic tensor into the canonical Lie tensor, with intriguing geometric and algebraic implications.

The *Hamilton-Jacobi-Santilli isodual equations* are then given by [4,9]

$$\partial^d A^{\circ d} /^d \partial^d t^d + H^d \ = \ 0, \tag{2.2.21a}$$

$$\partial^d A^{\circ d} /^d \partial^d x_k^d - p_k^d \ = \ 0, \ \partial^d A^{\circ d} /^d \partial^d p_k^d \equiv 0. \tag{2.2.21b}$$

The *Lie-Santilli isodual brackets* among two isodual functions A^d and B^d on $S^d(t^d, \, x^d, \, p^d)$ then become

$$[A^d ,^d B^d] \ = \ \frac{\partial^d A^d}{\partial^d b^{d\mu}} d \ \times^d \omega^{d\mu\nu} \times^d \frac{\partial^d B^d}{\partial^d b^{d\nu}} d \ = \ -[A, B], \tag{2.2.22}$$

where

$$\omega^{d\mu\nu} \ = \ (\omega_{\mu\nu}) \tag{2.2.23}$$

is the *Lie-Santilli isodual tensor* (that coincides with the conventional canonical tensor). The direct representation of isodual equations in first-order form is self-evident.

In summary, all properties of the isodual theory at the Newtonian level carry over at the level of isodual Hamiltonian mechanics.

2.2.7 Isodual Galilean Relativity

As it is well known, the Newtonian, Lagrangian and Hamiltonian treatment of matter are only the pre-requisites for the characterization of physical laws via basic relativities and their underlying symmetries. Therefore, no equivalence in the treatment of matter and antimatter can be achieved without identifying the relativities suitable for the *classical* treatment of antimatter.

To begin this study, we introduce the *Galilei-Santilli isodual symmetry* $G^d(3.1)$ [7,5,9,22-31] as the step-by-step isodual image of the conventional *Galilei symmetry* $G(3.1)$ (herein assumed to be known[2]). By using conventional symbols for the Galilean symmetry of a Keplerian system of N point particles with non-null masses $m_a,\ a = 1,\, 2, \ldots,\, n,\ G^d(3.1)$ is characterized by *isodual parameters and generators*

$$w^d \ = \ (\theta^d_k, r^{kd}_o, v^{kd}_o, t^d_o) = -w, \tag{2.2.24a}$$

$$J^d_k \ = \ \sum{}_{aijk} r^d_{ja} \times^d p^k_{ja} \ = \ -J_k \tag{2.2.24b}$$

$$P^d_k \ = \ \sum{}_a p^d_{ka} \ = \ -P_k, \tag{2.2.24c}$$

$$G^d_k \ = \ \sum{}_a (m^d_a \times^d r^d_{ak} - t^d \times p^d_{ak}), \tag{2.2.24d}$$

$$H^d \ = \ \frac{1}{2}^d \times^d \sum{}_a p^d_{ak} \times^d p^{kd}_a + V^d(r^d) \ = \ -H, \tag{2.2.24e}$$

equipped with the *isodual commutator*

$$[A^d,^d B^d] \ = \ \sum{}_{a,k} [(\partial^d A^d /^d \partial^d r^{kd}_a) \times^d (\partial^d B^d /^d \partial^d p^d_{ak}) -$$
$$- (\partial^d B^d /^d \partial^d r^{kd}_a) \times^d (\partial^d A^d /^d \partial^d p^d_{ak})]. \tag{2.2.25}$$

In accordance with rule (2.1.34), the structure constants and Casimir invariants of the isodual algebra $G^d(3.1)$ are negative-definite. If $g(w)$ is an element of the (connected component) of the Galilei group $G(3.1)$, its isodual is characterized by

$$g^d(w^d) \ = \ e^{d - i^d \times^d w^d \times^d X^d} = -e^{i \times (-w) \times X} \ = \ -g(-w) \in G^d(3.1). \tag{2.2.26}$$

The *Galilei-Santilli isodual transformations* are then given by

$$t^d \ \to \ t'^d \ = \ t^d + t^d_o \ = \ -t', \tag{2.2.27a}$$

$$r^d \ \to \ r'^d \ = \ r^d + r^d_o \ = \ -r' \tag{2.2.27b}$$

[2]The literature on the conventional Galilei and special relativities and related symmetries is so vast as to discourage discriminatory quotations.

$$r^d \;\rightarrow\; r'^d \;=\; r^d + v_o^d \times^d t_o^d \;=\; -r', \tag{2.2.27c}$$

$$r^d \;\rightarrow\; r'^d \;=\; R^d(\theta^d) \times^d r^d \;=\; -R(-\theta) \times r. \tag{2.2.27d}$$

where $R^d(\theta^d)$ is an element of the *isodual rotational symmetry* first studied in the original proposal [1].

The desired classical nonrelativistic characterization of antimatter is therefore given by imposing the $G^d(3.1)$ invariance to the considered isodual equations. This implies, in particular, that the equations admit a representation via isodual Lagrangian and Hamiltonian mechanics.

We now confirm the classical experimental verification of the above isodual representation of antimatter already treated in Section 2.2.2. Consider a conventional, classical, massive *particle* and its *antiparticle* in exterior dynamical conditions in vacuum. Suppose that the particle and antiparticle have charge $-e$ and $+e$, respectively (say, an *electron* and a *positron*), and that they enter into the gap of a magnet with constant magnetic field $\mathbf{B}$.

As it is well known, visual experimental observation establishes that particles and antiparticles under the same magnetic field have spiral trajectories of *opposite orientation*. But this behavior occurs for the *representation of both the particle and its antiparticle in the same Euclidean space*. The situation under isoduality is different, as described by the following:

LEMMA 2.2.1 [5a]: *The trajectories under the same magnetic field of a charged particle in Euclidean space and of the corresponding antiparticle in isodual Euclidean space coincide.*

Proof: Suppose that the particle has negative charge $-e$ in Euclidean space $E(r,\, \delta,\, R)$, i.e., the value $-e$ is defined with respect to the positive unit $+1$ of the underlying field of real numbers $R = R(n,\, +,\, \times)$. Suppose that the particle is under the influence of the magnetic field $\mathbf{B}$.

The characterization of the corresponding antiparticle via isoduality implies the reversal of the sign of all physical quantities, thus yielding the charge $(-e)^d = +e$ in the isodual Euclidean space $E^d(r^d,\, \delta^d,\, R^d)$, as well as the reversal of the magnetic field $B^d = -B$, although now defined with respect to the negative unit $(+1)^d = -1$.

It is then evident that the trajectory of a particle with charge $-e$ in the field B defined with respect to the unit $+1$ in Euclidean space and that for the antiparticle of charge $+e$ in the field $-B$ defined with respect to the unit -1 in isodual Euclidean space coincide (Figure 2.3). **q.e.d.**

An aspect of Lemma 2.2.1, which is particularly important for this monograph, is given by the following:

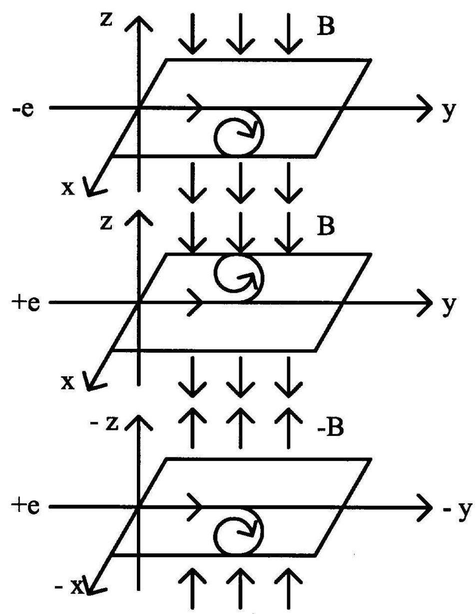

Figure 2.3. A schematic view of the trajectories of an electron and a positron with the same kinetic energy under the same magnetic field. The trajectories "appear" to be the reverse of each other when inspected by one observer, such as that in our spacetime (top and central views). However, when the two trajectories are represented in their corresponding spacetimes they coincide, as shown in the text (top and bottom views).

COROLLARY 2.2.1A: Antiparticles reverse their trajectories when projected from their own isodual space into our own space.

Lemma 2.2.1 assures that isodualities permit the representation of the correct trajectories of antiparticles as physically observed, despite their negative energy, thus providing the foundations for a consistent representation of antiparticles at the level of *first* quantization studied in the next section. Moreover, Lemma 2.2.1 tells us that the trajectories of antiparticles *appear* to exist in our space while in reality they belong to an independent space.

2.2.8 Isodual Special Relativity

We now introduce *isodual special relativity* for the classical relativistic treatment of point-like antiparticles (for the conventional case see Ref. [32]).

As it is well known, conventional special relativity is constructed on the fundamental 4-dimensional unit of the Minkowski space $I = Diag.(1, 1, 1, 1)$, representing the dimensionless units of space, e.g., $(+1\,\mathrm{cm}, +1\,\mathrm{cm}, +1\,\mathrm{cm})$, and the dimensionless unit of time, e.g., $+1$ sec, and constituting the basic unit of the conventional *Poincarè symmetry* $P(3.1)$ (hereon assumed to be known).

It then follows that *isodual special relativity* is characterized by the map

$$I \;=\; Diag.(\{1,\,1,\,1\},\,1) > 0 \rightarrow$$
$$rightarrow I^d \;=\; Diag.(\{-1,\,-1,\,-1\},\,-1) < 0. \qquad (2.2.28)$$

namely, the antimatter relativity is based on *negative units of space and time*, e.g., $I^d = Diag.(-1\,\mathrm{cm}, -1\,\mathrm{cm}, -1\,\mathrm{cm}, -1\,\mathrm{sec})$. This implies the reconstruction of the entire mathematics of the special relativity with respect to the common, isodual unit I^d, including: the *isodual field* $R^d = R^d(n^d,\, +^d,\, \times^d)$ of *isodual numbers* $n^d = n \times I^d$; the *isodual Minkowski spacetime* $M^d(x^d, \eta^d, R^d)$ with isodual coordinates $x^d = x \times I^d$, isodual metric $\eta^d = -\eta$ and basic invariant over R^d

$$(x-y)^{d2d} \;=\; [(x^\mu - y^\mu) \times \eta^d_{\mu\nu} \times (x^\nu - y^\nu)] \times I^d \in R^d. \qquad (2.2.29)$$

This procedure yields the central symmetry of this chapter indicated in Section 2.2.6, today known as the *Poincaré-Santilli isodual symmetry* [7]

$$P^d(3.1) \;=\; \mathcal{L}^d(3.1) \times^d T^d(3.1), \qquad (2.2.30)$$

where $\mathcal{L}^d(3.1)$ is the *Lorentz-Santilli isodual symmetry*, $\times^d$ is the *isodual direct product* and $T^d(3.1)$ represents the *isodual translations.*

The algebra of the connected component $P^{\uparrow d}_+(3.1)$ of $P^d(3.1)$ can be constructed in terms of the isodual parameters $w^d = \{-w_k\} = \{-\theta, -v, -a\}$ and isodual generators $X^d = -X = \{-X_k\} = \{-M_{\mu\nu}, -P_\mu\}$. The isodual commutator rules are given by [7]

$$[M^d_{\mu\nu},{}^d M_{\alpha\beta}]^d \;=$$
$$= i^d \times^d (\eta^d_{\nu\alpha} \times^d M^d_{\mu\beta} - \eta^d_{\mu\alpha} \times^d M^d_{\nu\beta} - \eta^d_{\nu\beta} \times^d M^d_{\mu\alpha} + \eta^d_{\mu\beta} \times^d \hat{M}^d_{\alpha\nu}), \qquad (2.2.31a)$$

$$[M^d_{\mu\nu},{}^d p^d_\alpha] \;=\; i^d \times^d (\eta^d_{\mu\alpha} \times^d p^d_\nu - \eta^d_{\nu\alpha} \times^d p^d_\mu), \qquad (2.2.31b)$$

$$[p^d_\alpha, p^d_\beta]^d \;=\; 0. \qquad (2.3.31c)$$

The *Poincarè-Santilli isodual transformations* are given by[3]

$$x^{1d\prime} \;=\; x^{1d} = -x^1, \qquad (2.2.32a)$$

$$x^{2d\prime} \;=\; x^{2d} = -x^2, \qquad (2.2.32b)$$

[3]It should be indicated that, contrary to popular beliefs, the conventional Poincaré symmetry will be shown in Chapter 3 to be *eleven* dimensional, the 11-th dimension being given by a new invariant under change of the unit. Therefore, the isodual symmetry $P^d(3.1)$ is also 11-dimensional.

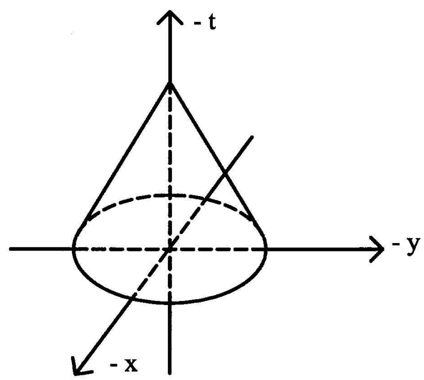

Figure 2.4. A schematic view of the "isodual backward light cone" as seen by an observer in our own spacetime with a time evolution reversed with respect to the "conventional forward light cone."

$$x^{3d\prime} = \gamma^d \times^d (x^{3d} - \beta^d \times^d x^{4d}) = -x^{3\prime}, \tag{2.2.32c}$$
$$x^{4d\prime} = \gamma^d \times^d (x^{4d} - \beta^d \times^d x^{3d}) = -x^{4\prime}, \tag{2.2.32d}$$
$$x^{d\mu\prime} = x^{d\mu} + a^{d\mu} = -x^{\mu\prime}, \tag{2.3.32e}$$

where

$$\beta^d = v^d /^d c_\circ^d = -\beta, \ \beta^{d2d} = -\beta^2, \ \gamma^d = -(1-\beta^2)^{-1/2}, \tag{2.2.33}$$

and the use of the isodual operations (quotient, square roots, etc.) is assumed.

The *isodual spinorial covering*

$$\mathcal{P}^d(3.1) = \mathcal{SL}^d(2.C^d) \times^d \mathcal{T}^d(3.1) \tag{2.2.34}$$

can then be constructed via the same methods.

The basic postulates of the isodual special relativity are also a simple isodual image of the conventional postulates [7]. For instance, the *maximal isodual causal speed in vacuum* is the speed of light in M^d, i.e.,

$$V_{max}^d = c_\circ^d = -c_\circ, \tag{2.2.35}$$

with the understanding that it is measured with a *negative-definite unit*, thus being fully equivalent to the conventional maximal speed c_o referred to a positive unit. A similar situation occurs for all other postulates.

The *isodual light cone* is evidently given by (Figure 2.4)

$$x^{d\,2\,d} = (x^{\mu d} \times^d \eta_{\mu\nu}^d \times^d x^{\nu d}) \times I^d =$$

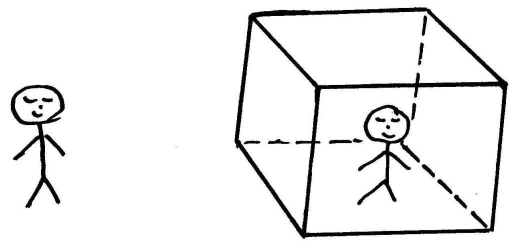

Figure 2.5. A schematic view of the "isodual cube," here defined as a conventional cube with two observers, an external observer in our spacetime and an internal observer in the isodual spacetime. The first implication of the isodual theory is that the same cube coexists in the two spacetimes and can, therefore, be detected by both observers. A most intriguing implication of the isodual theory is that each observer sees the other becoming younger. This occurrence is evident for the behavior of the internal observer with respect to the exterior one, since the former evolves according to a time opposite that of the latter. The same occurrence is less obvious for the opposite case, the behavior of the external observer with respect to the internal one, and it is due to the fact that the projection of our positive time into the isodual spacetime is indeed a motion backward in that spacetime.

$$= (-x \times x - y \times y - z \times z + t \times c_\circ^2 \times t) \times (-I) = 0. \tag{2.2.36}$$

As one can see, the above cone formally coincides with the conventional light cone, although the two cones belong to different spacetimes. The isodual light cone is used in these studies as *the cone of light emitted by antimatter in empty space (exterior problem).*

Note that the *two* Minkowskian metrics $\eta = Diag.(+1, +1, +1, -1)$ and $\eta = Diag.(-1, -1, -1, +1)$ have been popular since Minkowski's times, although both referred to the *same* unit I. We have learned here that these two popular metrics are connected by isoduality.

We finally introduce the *isodual electromagnetic waves* and related *isodual Maxwell's equations* [9]

$$F^d_{\mu\nu} = \partial^d A^d_\mu /^d \partial^d x^{\nu d} - \partial^d A^d_\nu /^d \partial^d x^{d\mu}, \tag{2.2.37a}$$

$$\partial^d_\lambda F^d_{\mu\nu} + \partial^d_\mu F^d_{\nu\lambda} + \partial^d_\nu F^d_{\lambda\mu} = 0, \tag{2.2.37b}$$

$$\partial^d_\mu F^{d\mu\nu} = -J^{d\nu}. \tag{2.2.37c}$$

As we shall see, the nontriviality of the isodual special relativity is illustrated by the fact that isodual electromagnetic waves experience gravitational repulsion when in the field of matter.

2.2.9 Inequivalence of Isodual and Spacetime Inversions

As it is well known (see, the fundamental spacetime symmetries of the 20-th century are the continuous (connected) component of the Poincaré symmetry plus discrete symmetries characterized by *space reversal* (also called *parity*) and *time reversal.*

As noted earlier, antiparticles are assumed in the above setting to exist in the same representation spacetime and to obey the same symmetries as those of particles. On the contrary, according to the isodual theory, antiparticles are represented in a spacetime and possess symmetries distinct from those of particles, although connected to the latter by the isodual transform.

The latter occurrence requires the introduction of the *isodual spacetime inversions,* that is, the isodual images of space and time inversions, first identified in Ref. [9], that can be formulated in unified coordinate form as follows

$$\begin{aligned} x^{d\mu\prime} \; = \; \pi^d \times^d x^d \; = \; -\pi \times x = \\ = (-r, x^4), \;\; \tau^d \times^d x^d = -\tau \times x \; = \; -(r, -x^4), \end{aligned} \tag{2.2.38}$$

with field theoretical extension (here expressed for simplicity for a scalar field)

$$\pi^d \times^d \phi^d(x^d) \times^d \pi^{d\dagger} = \phi^d(x^{\prime d}, x^{\prime d} = (-r^d, t^d) = (r, -t), \tag{2.2.39a}$$

$$\tau^d \times^d \phi^d(x^d) \times^d \tau^{d\dagger} = \bar{\phi}^d(x^{\prime\prime d}, x^{\prime\prime d} = (r^d, -t^d) = (-r, t), \tag{2.2.39b}$$

where $r^d(=-r)$ is the *isodual coordinate* on space $E^d(r^d,\, \delta^d,\, R^d)$, and t^d is the *isodual time* on $E^d(t^d,\, 1,\, R^d_t)$.

LEMMA 2.2.2 [9]: *Isodual inversions and spacetime inversions are inequivalent.*

Proof. Spacetime inversions are characterized by the change of sign $x \rightarrow -x$ *by always preserving the original metric measured with positive units*, while isodual inversions imply the map $x \rightarrow x^d = -x$ *but now measured with an isodual metric* $\eta^d = -\eta$ *with negative units* $I^d = -I$, thus being inequivalent. **q.e.d.**

Despite their simplicity, isodual inversions (or isodual discrete symmetries) are not trivial (Figure 2.6). In fact, all measurements are done in our spacetime, thus implying the need to consider the *projection* of the isodual discrete symmetries into our spacetime which are manifestly different than the conventional forms.

In particular, they imply a sort of interchange, in the sense that the conventional *space* inversion $(r, t) \rightarrow (-r, t)$ emerges as belonging to the projection in our spacetime of the isodual *time* inversion, and vice-versa.

Note that the above "interchange" of parity and time reversal of isodual particles projected in our spacetime could be used for experimental verifications, but this aspect is left to interested readers.

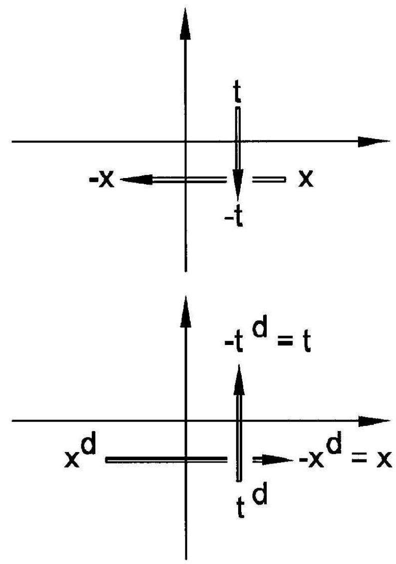

Figure 2.6. A schematic view of the additional peculiar property that the projection in our spacetime of the isodual space inversion appears as a time inversion and vice versa. In fact, a point in the isodual spacetime is given by $(x^d, t^d) = (-x, -t)$. The projection in our spacetime of the isodual space inversion $(x^d, t^d) \rightarrow (-x^d, t^d)$ is then given by $(x, -t)$, thus appearing as a time (rather than a space) inversion. Similarly, the projection in our spacetime of the isodual time inversion $(x^d, t^d) \rightarrow (x^d, -t^d)$ appears as $(-x, t)$, that is, as a space (rather than time) inversion. Despite its simplicity, the above occurrence has rather deep implications for all discrete symmetries in particle physics indicated later on.

In closing this subsection, we point out that the notion of isodual parity has intriguing connections with the parity of antiparticles in the $(j, 0) + (0, j)$ representation space more recently studied by Ahluwalia, Johnson and Goldman [10]. In fact, the latter parity results in being opposite that of particles which is fully in line with isodual space inversion (isodual parity).

2.2.10 Dunning-Davies Isodual Thermodynamics of Antimatter

An important contribution to the isodual theory has been made by J. Dunning-Davies [11] who introduced in 1999 the first, and only known consistent thermodynamics for antimatter, here called *Dunning-Davies antimatter thermodynamics* with intriguing results and implications.

As conventionally done in the field, let us represent heat with Q, internal energy with U, work with W, entropy with S, and absolute temperature with T. *Dunning-Davies isodual thermodynamics of antimatter* is evidently defined via the isodual quantities

$$Q^d = -Q,\ U^d = -U,\ W^d = -W,\ S^d = -S,\ T^d = -T \tag{2.2.40}$$

on isodual spaces over the isodual field of real numbers $R^d = R^d(n^d, +^d, \times^d)$ with isodual unit $I^d = -1$.

Recall from Section 2.1.3 that *differentials are isoselfdual* (that is, invariant under isoduality). Dunning-Davies then has the following:

THEOREM 2.2.1 [21]: *Thermodynamical laws are isoselfdual.*

Proof. For the *First Law of thermodynamics* we have

$$dQ = dU - dW \ \equiv\ d^dQ^d = d^dU^d - d^dW^d. \tag{2.2.41}$$

Similarly, for the *Second Law of thermodynamics* we have

$$dQ = T \times dS \ \equiv\ d^dQ^d = T^d \times^d S^d, \tag{2.2.42}$$

and the same occurs for the remaining laws. **q.e.d.**

Despite their simplicity, Dunning-Davies results [21] have rather deep implications. First, the identity of thermodynamical laws, by no means, implies the identity of the thermodynamics of matter and antimatter. In fact, *in Dunning-Davies isodual thermodynamics the entropy must always decrease in time*, since the isodual entropy is always negative and is defined in a space with evolution backward in time with respect to us. However, these features are fully equivalent to the conventional increase of the entropy tacitly referred to positive units.

Also, Dunning-Davies results indicate that *antimatter galaxies and quasars cannot be distinguished from matter galaxies and quasars via the use of thermodynamics,* evidently because their laws coincide, in a way much similar to the identity of the trajectories of particles and antiparticles of Lemma 2.2.1.

This result indicates that the only possibility known at this writing to determine whether far-away galaxies and quasars are made up of matter or of antimatter is that via the predicted gravitational repulsion of the light emitted by antimatter called *isodual light* (see next section and Chapter 5).

2.2.11 Isodual General Relativity

For completeness, we now introduce the *isodual general relativity* for the classical gravitational representation of antimatter. A primary motivation for its study is the incompatibility with antimatter of the positive-definite character of the energy-momentum tensor of the conventional general relativity studied in Chapter 1.

The resolution of this incompatibility evidently requires a structural revision of general relativity [33] for a consistent treatment of antimatter. The *only* solution known to the author is that offered by isoduality.[4]

It should be stressed that this study is here presented merely for completeness, since the achievement of a consistent treatment of negative-energies, by no means, resolves the serious inconsistencies of gravitation on a Riemannian space caused by curvature, as studied in Section 1.2, thus requiring new geometric vistas beyond those permitted by the Riemannian geometry (see Chapters 3 and 4).

As studied in Section 2.1.7, the *isodual Riemannian geometry* is defined on the isodual field $R^d(n^d, +^d, \times^d)$ for which *the norm is negative-definite*, Eq. (2.1.18). As a result, *all quantities that are positive in Riemannian geometry become negative under isoduality, thus including the energy-momentum tensor.*

In fact, the energy-momentum tensor of isodual electromagnetic waves (2.2.37) is negative-definite [8,9]

$$T^d_{\mu\nu} \;=\; (4\times\pi)^{-1d}\times^d(F^d_{\mu\alpha}\times^d F^d_{\alpha_\nu} + (1/4)^{-1d}\times^d g^d_{\mu\nu}\times^d F^d_{\alpha\beta}\times^d F^{d\alpha\beta}). \quad (2.2.43)$$

The *Einstein-Hilbert isodual equations for antimatter in the exterior conditions in vacuum* are then given by [6,9]

$$G^d_{\mu\nu} \;=\; R^d_{\mu\nu} - \frac{1}{2}^d \times^d g^d_{\mu\nu}\times^d R^d \;=\; k^d\times^d T^d_{\mu\nu}. \quad (2.2.44)$$

The rest of the theory is then given by the use of the isodual Riemannian geometry of Section 2.1.7.

The explicit study of this gravitational theory of antimatter is left to the interested reader due to the indicated inconsistencies of gravitational theories on a Riemannian space for the *conventional case of matter* (Section 1.2). These inconsistencies multiply when treating antimatter, as we shall see.

2.3 OPERATOR ISODUAL THEORY OF POINT-LIKE ANTIPARTICLES

2.3.1 Basic Assumptions

In this section we study the operator image of the classical isodual theory of the preceding section; we prove that the operator image of isoduality is equivalent to charge conjugation; and we show that isodual mathematics resolves all known objections against negative energies.

A main result of this section is the identification of a simple, structurally new formulation of quantum mechanics known as *isodual quantum mechanics* or, more

[4]The author would be grateful to colleagues who care to bring to his attention other "classical" gravitational theories of antimatter compatible with the negative-energy solutions needed by antimatter.

properly, as the *isodual branch of hadronic mechanics* first proposed by Santilli in Refs. [5]. Another result of this section is the fact that all numerical predictions of operator isoduality coincide with those obtained via charge conjugation on a Hilbert space, thus providing the experimental verification of the isodual theory of antimatter at the operator level.

Despite that, the isodual image of quantum mechanics is not trivial because of a number of far reaching predictions we shall study in this section and in the next chapters, such as: the prediction that antimatter emits a new light distinct from that of matter; antiparticles in the gravitational field of matter experience antigravity; bound states of particles and their antiparticles can move backward in time without violating the principle of causality; and other predictions.

Other important results of this section are a new interpretation of the conventional Dirac equation that escaped detection for about one century, as well as the indication that the isodual theory of antimatter originated from the Dirac equation itself, not so much from the negative-energy solutions, but more properly from their two-dimensional unit that is indeed negative-definite, $I_{2\times 2} = Diag.(-1,-1)$.

As we shall see, Dirac's "hole theory", with the consequential restriction of the study of antimatter to the sole second quantization and resulting scientific imbalance indicated in Section 1.1, were due to Dirac's lack of knowledge of a *mathematics based on negative units.*

Intriguingly, had Dirac identified the quantity $I_{2\times 2} = Diag.(-1,-1)$ as the unit of the *mathematics* treating the negative energy solutions of his equation, the physics of the 20-th century would have followed a different path because, despite its simplicity, the unit is indeed the most fundamental notion of all mathematical and physical theories.

2.3.2 Isodual Quantization

The isodual Hamiltonian mechanics (and its underlying *isodual symplectic geometry* [5a] not treated in this chapter for brevity) permit the identification of a new quantization channel, known as the *naive isodual quantization* [6] that can be readily formulated via the use of the Hamilton-Jacobi-Santilli isodual equations (2.2.21) as follows

$$A^{\circ d} \rightarrow -i^d \times^d \hbar^d \times^d Ln^d \psi^d(t^d, r^d), \tag{2.3.1a}$$

$$\begin{aligned} \partial^d A^{\circ d}/^d \partial^d t^d + H^d \ = \ 0 \rightarrow i^d \times^d \partial^d \psi^d /^d \partial^d t^d \ = \\ = \ H^d \times^d \psi^d \ = \ E^d \times^d \psi^d, \end{aligned} \tag{2.3.1b}$$

$$\partial^d A^{\circ d}/^d \partial^d x^{dk} - \hat{p}_k \ = \ 0 \rightarrow p^d_k \times^d \psi^d \ = \ -i^d \times^d \partial^d_k \psi^d, \tag{2.3.1c}$$

$$\partial^d A^{\circ d}/^d \partial^d p^d_k = 0 \rightarrow \partial^d \psi^d /^d \partial^d p^d_k = 0. \tag{2.3.1d}$$

Recall that the fundamental unit of quantum mechanics is Planck's constant $\hbar = +1$. It then follows that the fundamental unit of the isodual operator theory is the new quantity

$$\hbar^d = -1. \tag{2.3.2}$$

It is evident that the above quantization channel identifies the new mechanics known as *isodual quantum mechanics,* or the *isodual branch of hadronic mechanics.*

2.3.3 Isodual Hilbert Spaces

Isodual quantum mechanics can be constructed via the anti-unitary transform

$$U \times U^\dagger = \hbar^d = I^d = -1, \tag{2.3.3}$$

applied, for consistency, to the *totality* of the mathematical and physical formulations of quantum mechanics. We recover in this way the isodual real and complex numbers

$$n \rightarrow n^d = U \times n \times U^\dagger = n \times (U \times U^\dagger) = n \times I^d, \tag{2.3.4}$$

isodual operators

$$A \rightarrow U \times A \times U^\dagger = A^d, \tag{2.3.5}$$

the isodual product among generic quantities A, B (numbers, operators, etc.)

$$A \times B \rightarrow U \times (A \times B) \times U^\dagger =$$
$$= (U \times A \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times B \times U^\dagger) = A^d \times^d B^d, \tag{2.3.6}$$

and similar properties.

Evidently, isodual quantum mechanics is formulated in the *isodual Hilbert space* $\mathcal{H}^d$ with *isodual states* [6]

$$|\psi>^d = -|\psi>^\dagger = -<\psi|, \tag{2.3.7}$$

where $<\psi|$ is a conventional dual state on $\mathcal{H}$, and *isodual inner product*

$$<\psi|^d \times (-1) \times |\psi>^d \times I^d, \tag{2.3.8}$$

with *isodual expectation values* of an operator A^d

$$<A^d>^d = (<\psi|^d \times^d A^d \times^d |\psi>^d /^d <\psi|^d \times^d |\psi>^d), \tag{2.3.9}$$

and *isodual normalization*

$$<\psi|^d \times^d |\psi>^d = -1 \tag{2.3.10}$$

defined on the *isodual complex field* C^d with unit -1 (Section 2.1.1).

The isodual expectation values can also be reached via anti-unitary transform (2.3.3),

$$< \psi| \times A \times |\psi > \rightarrow U \times (< \psi| \times A \times |\psi >) \times U^\dagger =$$
$$= (< \psi| \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times A \times U^\dagger) \times (U \times U^\dagger)^{-1} \times$$
$$\times (U \times |\psi >) \times (U \times U^\dagger) = < \psi|^d \times^d A^d \times^d |\psi >^d \times I^d. \tag{2.3.11}$$

The proof of the following property is trivial.

LEMMA 2.3.1 [5b]: *The isodual image of an operator A that is Hermitian on $\mathcal{H}$ over C is also Hermitian on $\mathcal{H}^d$ over C^d (isodual Hermiticity).*

It then follows that *all quantities that are observables for particles are equally observables for antiparticles represented via isoduality.*

LEMMA 2.3.2 [5b]: *Let H be a Hermitian operator on a Hilbert space $\mathcal{H}$ over C with positive-definite eigenvalues E,*

$$H \times |\psi > = E \times |\psi >, H = H^\dagger, E => 0. \tag{2.3.12}$$

Then, the eigenvalues of the isodual operator H^d on the isodual Hilbert space $\mathcal{H}^d$ over C^d are negative-definite,

$$H^d \times^d |\psi >^d = E^d \times^d |\psi >^d, H^d = H^{d\dagger^d}, E^d < 0. \tag{2.3.13}$$

This important property establishes an evident compatibility between the classical and operator formulations of isoduality.

We also mention the *isodual unitary laws*

$$U^d \times^d U^{d\dagger} = U^{d\dagger} \times^d U^d = I^d, \tag{2.3.14}$$

the *isodual trace*

$$Tr^d A^d = (Tr A^d) \times I^d \in C^d, \tag{2.3.15a}$$
$$Tr^d (A^d \times^d B^d) = Tr^d A^d \times^d Tr^d B^d, \tag{2.3.15b}$$

the *isodual determinant*

$$Det^d A^d = (Det A^d) \times I^d \in C^d, \tag{2.3.16a}$$
$$Det^d (A^d \times^d B^d) = Det^d \times^d Det^d B^d, \tag{2.3.16b}$$

the *isodual logarithm* of a real number n

$$Log^d n^d = -(Log\, n^d) \times I^d, \tag{2.3.17}$$

and other isodual operations.

The interested reader can then work out the remaining properties of the isodual theory of linear operators on a Hilbert space.

2.3.4 Isoselfduality of Minkowski's Line Elements and Hilbert's Inner Products

A most fundamental new property of the isodual theory, with implications as vast as the formulation of a basically new cosmology, is expressed by the following lemma whose proof is a trivial application of transform (2.3.3).

LEMMA 2.3.3 [23]: *Minkowski's line elements and Hilbert's inner products are invariant under isoduality (or they are isoselfdual according to Definition 2.1.2),*

$$x^2 = (x^\mu \times \eta_{\mu\nu} \times x^\nu) \times I \equiv$$

$$\equiv \ (x^{d\mu} \times^d \eta^d_{\mu\nu} \times^d x^{d\nu}) \times I^d = x^{d2^d}, \tag{2.3.18a}$$

$$< \psi| \times |\psi > \times I \equiv \ < \psi|^d \times^d |\psi >^d \times I^d. \tag{2.3.18b}$$

As a result, *all relativistic and quantum mechanical laws holding for matter also hold for antimatter under isoduality.* The equivalence of charge conjugation and isoduality then follows, as we shall see shortly.

Lemma 2.3.3 illustrates the reason why isodual special relativity and isodual Hilbert spaces have escaped detection for about one century. Note, however, that invariances (2.3.18) require the prior discovery of *new numbers,* those with negative unit.

2.3.5 Isodual Schrödinger and Heisenberg's Equations

The fundamental dynamical equations of isodual quantum mechanics are the isodual images of conventional dynamical equations. They are today known as the *Schrödinger-Santilli isodual equations* [4] (where we assume hereon $\hbar^d = -1$, thus having $\times^d \hbar^d = 1$)

$$i^d \times^d \partial|\psi >^d /^d \partial^d t^d = H^d \times^d |\psi >^d, \tag{2.3.19a}$$

$$p^d_k \times^d |\psi >^d = -i^d \times^d \partial^d |\psi >^d /^d \partial^d r^d, \tag{2.3.19b}$$

and the *Heisenberg-Santilli isodual equations*

$$i^d \times^d d^d A^d /^d d^d t^d = A^d \times^d H^d - H^d \times^d A^d = [A^d, H^d]^d, \tag{2.3.20a}$$

$$[r^d_i, p^d_j]^d = i^d \times^d \delta^{di}_j, [r^d, r^{dj}]^d = [p^d_i, p^d_j]^d = 0. \tag{2.3.20b}$$

Note that, when written explicitly, Eq. (2.3.19a) is based on an associative modular action *to the left,*

$$- < \psi| \times^d H^d = (\partial^d < \psi|\partial^d t^d) \times^d i^d. \tag{2.3.21}$$

It is an instructive exercise for readers interested in learning the new mechanics to prove the equivalence of the isodual Schrödinger and Heisenberg equations via the anti-unitary transform (2.3.3).

2.3.6 Isoselfdual Re-Interpretation of Dirac's Equation

Isoduality has permitted a novel interpretation of the conventional *Dirac equation* (we shall here used the notation of Ref. [12]) in which the negative-energy states are reinterpreted as belonging to the isodual images of positive energy states, resulting in the first known *consistent representation of antiparticles in first quantization.*

This result should be expected since the isodual theory of antimatter applies at the Newtonian level, let alone that of first quantization. Needless to say, the treatment via isodual first quantization does not exclude that via isodual second quantization. The point is that the treatment of antiparticles is no longer restricted to second quantization, as a condition to resolve the scientific imbalance between matter and antimatter indicated earlier.

Consider the conventional Dirac equation [2]

$$[\gamma^\mu \times (p_\mu - e \times A_\mu/c) + i \times m] \times \Psi(x) = 0, \tag{2.3.22}$$

with realization of Dirac's celebrated gamma matrices

$$\gamma_k = \begin{pmatrix} 0 & -\sigma_k \\ \sigma_k & 0 \end{pmatrix}, \quad \gamma^4 = i \times \begin{pmatrix} I_{2\times 2} & 0, \\ 0 & -I_{2\times 2} \end{pmatrix}, \tag{2.3.23a}$$

$$\{\gamma_\mu, \tilde{\gamma}_\nu\} \ = \ 2\times\eta_{\mu\nu}, \quad \Psi = i \times \begin{pmatrix} \Phi \\ -\Phi^\dagger \end{pmatrix}. \tag{2.3.23b}$$

At the level of first quantization here considered, the above equation is rather universally interpreted as representing an electron under an external electromagnetic field.

The above equations are generally defined in the 6-dimensional space given by the Kronecker product of the conventional Minkowski spacetime and an internal spin space

$$M_{Tot} = M(x, \eta, R) \times S_{spin}, \tag{2.3.24}$$

with total unit

$$I_{Tot} = I_{orb} \times I_{spin} = Diag.(1,\ 1,\ 1,\ 1) \times Diag.(1,\ 1), \tag{2.3.25}$$

and total symmetry

$$P(3.1) = SL(2.C) \times T(3.1). \tag{2.3.26}$$

The proof of the following property is recommended to interested readers.

THEOREM 2.3.1 [5b]: *Pauli's sigma matrices and Dirac's gamma matrices are isoselfdual*,

$$\sigma_k \equiv \sigma^d_k, \tag{2.3.27a}$$

$$\gamma_\mu \equiv \gamma^d_\mu. \tag{2.3.27b}$$

The above properties imply an important re-interpretation of Eq. (2.3.22), first identified in Ref. [9] and today known as the *Dirac-Santilli isoselfdual equation,* that can be written

$$[\tilde{\gamma}^{\mu} \times (p_{\mu} - e \times A_{\mu}/c) + i \times m] \times \tilde{\Psi}(x) = 0, \tag{2.3.28}$$

with re-interpretation of the gamma matrices

$$\tilde{\gamma}_k = \begin{pmatrix} 0 & \sigma_k^d \\ \sigma_k & 0 \end{pmatrix}, \quad \tilde{\gamma}^4 = i \begin{pmatrix} I_{2\times 2} & 0, \\ 0 & I^d_{2\times 2} \end{pmatrix}, \tag{2.3.29a}$$

$$\{\tilde{\gamma}_{\mu}, \tilde{\gamma}_{\nu}\} = 2^d \times^d \eta^d_{\mu\nu}, \quad \tilde{\Psi} = -\tilde{\gamma}_4 \times \Psi = i \times \begin{pmatrix} \Phi \\ \Phi^d \end{pmatrix}, \tag{2.3.29b}$$

By recalling that isodual spaces coexist with, but are different from conventional spaces, we have the following:

THEOREM 2.3.2 [9]: *The Dirac-Santilli isoselfdual equation is defined on the 12-dimensional isoselfdual representation space*

$$M_{Tot} = \{M(x, \eta, R) \times S_{spin}\} \times \{M^d(x^d, \eta^d, R^d) \times^d S^d_{spin}\}, \tag{2.3.30}$$

with isoselfdual total 12-dimensional unit

$$I_{Tot} = \{I_{orb} \times I_{spin}\} \times \{I^d_{orb} \times^d I^d_{spin}\}, \tag{2.3.31}$$

and its symmetry is given by the isoselfdual product of the Poincaré symmetry and its isodual

$$S_{Tot} = \mathcal{P}(3.1) \times \mathcal{P}^d(3.1) =$$
$$= \{SL(2.C) \times T(3.1)\} \times \{SL^d(2.C^d) \times^d T^d(3.1)\}. \tag{2.3.32}$$

A direct consequence of the isoselfdual structure can be expressed as follows.

COROLLARY 2.3.2a [9]: *The Dirac-Santilli isoselfdual equation provides a joint representation of an electron and its antiparticle (the positron) in first quantization,*

$$Dirac\ \ Equation = Electron \times Positron. \tag{2.3.33}$$

In fact, the two-dimensional component of the wave function with positive-energy solution represents the electron and that with negative-energy solutions represent the positron without any need for second quantization, due to the physical behavior of negative energies in isodual treatment established earlier.

Note the complete democracy and equivalence in treatment of the electron and the positron in equation (2.3.28), in the sense that the equation can be equally used to represent an electron or its antiparticle. By comparison, according to the original Dirac interpretation, the equation could only be used to represent the electron [12], since the representation of the positron required the "hole theory".

It has been popularly believed throughout the 20-th century that Dirac's gamma matrices provide a "four-dimensional representation of the SU(2)-spin symmetry". This belief is disproved by the isodual theory, as expressed by the following

THEOREM 2.3.3 [5b]: *Dirac's gamma matrices characterize the direct product of an irreducible two-dimensional (regular) representation of the SU(2)-spin symmetry and its isodual,*

$$Dirac's\ \ Spin\ \ Symmetry : SU(2) \times SU^d(2). \tag{2.3.34}$$

In fact, the gamma matrices are characterized by the conventional, 2-dimensional Pauli matrices σ_k and related identity $I_{2\times 2}$ as well as other matrices that have resulted in being the exact isodual images σ_k^d with isodual unit $I_{2\times 2}^d$.

It should be recalled that the isodual theory was born precisely out of these issues and, more particularly, from the incompatibility between the popular interpretation of gamma matrices as providing a "four-dimensional" representation of the SU(2)-spin symmetry and the *lack of existence of such a representation in Lie's theory.*

The sole possibility known to the author for the reconciliation of Lie's theory for the SU(2)-spin symmetry and Dirac's gamma matrices was to assume that $-I_{2\times 2}$ is the unit of a dual-type representation. The entire theory studied in this chapter then followed.

It should also be noted that, as conventionally written, Dirac's equation *is not* isoselfdual because it is not sufficiently symmetric in the two-dimensional states and their isoduals.

In summary, Dirac's was forced to formulate the "hole theory" for antiparticles because he referred the *negative* energy states to the conventional *positive* unit, while their reformulation with respect to *negative* units yields fully physical results.

It is easy to see that the same isodual reinterpretation applies for Majorana's spinorial representations [13] (see also [14,15]) as well as Ahluwalia's broader spinorial representations (1/2, 0) + (0, 1/2) [16] (see also the subsequent paper [17]), that are reinterpreted in the isoselfdual form $(1, 2, 0) + (1, 2, 0)^d$, thus extending their physical applicability to first quantization.

In the latter reinterpretation the representation (1/2, 0) is evidently done conventional spaces over conventional fields with unit $+1$, while the isodual representation $(1/2, 0)^d$ is done on the corresponding isodual spaces defined on isodual fields with unit -1. As a result, all quantities of the representation (1/2, 0) change sign under isoduality.

It should be finally indicated that Ahluwalia treatment of Majorana spinors has a deep connection with isoduality because the underlying Class II spinors have a *negative norm* [16] precisely as it is the case for isoduality. As a result, the isodual reinterpretation under consideration here is quite natural and actually warranted for mathematical consistency, e.g., to have the topology characterized by a negative norm be compatible with the underlying fields.

2.3.7 Equivalence of Isoduality and charge conjugation

We come now to another fundamental point of this chapter, the proof that isoduality is equivalent to charge conjugation. This property is crucial for the experimental verification of isoduality at the particle level too. This equivalence was first identified by Santilli in Ref. [6] and can be easily expressed today via the following:

LEMMA 2.3.4 [6,5b,18]: *The isodual transform is equivalent to charge conjugation.*

Proof. Charge conjugation is characterized by the following transform of wavefunctions (see, e.g., Ref. [12], pages 109 and 176)

$$\Psi(x) \rightarrow C\Psi(x) = c \times \Psi^{\dagger}(x), \tag{2.3.35}$$

where

$$|c| = 1, \tag{2.3.36}$$

thus being manifestly equivalent to the isodual transform

$$\Psi(x) \rightarrow \Psi^{d}(x^{d}) = -\Psi^{\dagger}(-x^{t}), \tag{2.3.37}$$

where t denotes transpose.

A reason why the two transforms are equivalent, rather than identical, is the fact that charge conjugation maps spacetime into itself, while isoduality maps spacetime into its isodual. **q.e.d.**

Let us illustrate Lemma 2.3.4 with a few examples. As well known, the Klein-Gordon equation for a free particle

$$\partial^{\mu}\partial_{\mu}\Psi - m^{2} \times \Psi = 0 \tag{2.3.38}$$

is invariant under charge conjugation, in the sense that it is turned into the form

$$c \times [\bar{\Psi}\partial^{\mu}\partial_{\mu} - \bar{\Psi} \times m^{2}] = 0, \ \ |c| = 1, \tag{2.3.39}$$

where the upper bar denotes complex conjugation (since $\bar{\Psi}$ is a scalar), while the Lagrangian density

$$L = -(\hbar \times \hbar/2 \times m) \times \{\partial^{\mu}\bar{\Psi} - i \times e \times A^{\mu}/\hbar \times c) \times \bar{\Psi}]\times$$

$$\times[\partial\Psi + (i \times e \times A_\mu/\hbar \times c) \times \Psi] + m \times m \times \bar{\Psi} \times \Psi \tag{2.3.40}$$

is left invariant, and the four-current

$$J_\mu = -(i \times \hbar/2 \times m) \times [\bar{\psi} \times \partial_\mu\Psi - (\partial_\mu\bar{\Psi}) \times \Psi] \tag{2.3.41}$$

changes sign

$$J_\mu \to CJ_\mu = -J_\mu. \tag{2.3.42}$$

By recalling the selfduality of ordinary derivatives, Eq. (2.1.30), under isoduality the Klein-Gordon Equation becomes

$$[\partial^\mu\partial_\mu\Psi - m^2 \times \Psi]^d = \Psi^d\partial^{d\mu}\partial^d_\mu - \Psi^d \times^d m^d \times^d m^d =$$

$$= -[\bar{\Psi}\partial^\mu\partial_\mu - \bar{\Psi} \times m^2] = 0, \tag{2.3.43}$$

thus being equivalent to Eq. (2.3.39), while the Lagrangian changes sign and the four-current changes sign too,

$$J^d_\mu = -(i \times \hbar/2 \times m) \times [\bar{\Psi} \times \partial_\mu\Psi - (\partial_\mu\bar{\Psi}) \times \Psi]^d =$$

$$= (i \times \hbar/2 \times m) \times [\bar{\Psi} \times \partial_\mu\Psi - (\partial_\mu\bar{\Psi}) \times \Psi], \tag{2.3.44}$$

(where we have used the isoselfduality of the imaginary number i).

The above results confirm Lemma 2.3.4 because of the equivalent behavior of the equations of motion and the four-current, while the change of sign of the Lagrangian does not affect the numerical results.

As it is also well known, the Klein-Gordon equation for a particle under an external electromagnetic field [12]

$$[(\partial_\mu + i \times e \times A_\mu/\hbar \times c)\times$$

$$\times(\partial^\mu + i \times e \times A^\mu/\hbar \times c) - m^2] \times \Psi = 0, \tag{2.3.45}$$

is equally invariant under charge conjugation in which *either* e *or* A_μ change sign, in view of the known invariance

$$C(i \times e \times A_\mu/\hbar \times c) = i \times e \times A_\mu/\hbar \times c, \tag{2.3.46}$$

while the four-current also changes sign. By noting that the preceding invariance persists under isoduality,

$$(i \times e \times A_\mu/\hbar \times c)^d = i \times e \times A_\mu/\hbar \times c, \tag{2.3.47}$$

Eq. (2.3.45) remains invariant under isoduality, while the Lagrangian density changes sign and the four-current, again, changes sign.

Similarly, consider Dirac equation (see also Ref. [12], pp. 176-177)

$$[\gamma^\mu \times (\partial_\mu\Psi - (i \times e \times A_\mu/\hbar \times c) \times \Psi + m \times \Psi = 0, \tag{2.3.48}$$

with Lagrangian density

$$L = (\hbar \times c/2) \times \{\tilde{\Psi} \times \gamma^\mu \times [\partial_\mu \Psi + (i \times e \times A_\mu/\hbar \times c) \times \Psi] -$$
$$-(\partial^\mu \tilde{\Psi} - (i \times e \times A^\mu/\hbar \times c) \times \tilde{\Psi}] \times \gamma_\mu - m \times \tilde{\Psi} \times \Psi, \tag{2.3.49a}$$
$$\tilde{\Psi} = \Psi^\dagger \times \gamma_4, \tag{2.3.49b}$$

and four-current

$$J_\mu = i \times c \times \tilde{\Psi} \times \gamma_\mu \times \Psi = i \times c \times \Psi^\dagger \times \gamma_4 \times \gamma_\mu \times \Psi. \tag{2.3.50}$$

The charge conjugation for Dirac's equations is given by the transform [12]

$$\Psi \rightarrow C\Psi = c \times S_C^{-1} \times \tilde{\Psi}^t \tag{2.3.51}$$

where S_C is a unitary matrix such that

$$\gamma_\mu \rightarrow -\gamma_\mu^t = S_C \times \gamma_\mu \times S_C^{-1}, \tag{2.3.52}$$

and there is the change of sign *either* of e *or* of A_μ, under which the equation is transformed into the form

$$[\partial_\mu \tilde{\Psi} - (i \times e \times A_\mu/\hbar \times c) \times \tilde{\Psi}] \times \gamma^\mu - m \times \tilde{\Psi} = 0, \tag{2.3.53}$$

while the Lagrangian density changes sign and the four-current remains the same,

$$L \rightarrow CL = -L, \quad J_\mu \rightarrow CJ_\mu = J_\mu. \tag{2.3.54}$$

It is easy to see that isoduality provides equivalent results. In fact, we have for Eq. (2.3.48)

$$\{[\gamma^\mu \times (\partial_\mu \Psi - i \times e \times A_\mu/\hbar \times c) \times \Psi + m \times \Psi\}^d =$$
$$= [\partial_\mu \Psi^\dagger - (i \times e \times A_\mu/\hbar \times c) \times \Psi^\dagger] \times \gamma^\mu - m \times \Psi^\dagger = 0, \tag{2.3.55}$$

that, when multiplied by γ_4 reproduces Eq. (2.3.53) identically. Similarly, by recalling that Dirac' s gamma matrices are isoselfdual (Theorem 2.3.1), and by noting that

$$\tilde{\Psi}^d = (\Psi^\dagger \times \gamma_4)^d = \gamma_4 \times \Psi, \tag{2.3.56}$$

we have

$$L^d = L, \tag{2.3.57}$$

while for the four-current we have

$$J_\mu^d = -i \times c \times \Psi^\dagger \times \gamma_\mu \times \gamma_4 \times \psi. \tag{2.3.58}$$

But the γ_μ and γ_4 anticommutate. As a consequence, the four-current does not change sign under isoduality as in the conventional case.

Note that the lack of change of sign under isoduality of Dirac's four-current J_μ confirms reinterpretation (2.3.28) since, for the latter equation, the total charge is null.

The equivalence between isoduality and charge conjugation of other equations, such as those by Weyl, Majorana, etc., follows the same lines.

2.3.8 Experimental Verification of the Isodual Theory of Antimatter in Particle Physics

In Section 2.2.3. we have established the experimental verification of the isodual theory of antimatter in classical physics that, in particle physics, requires no detailed elaboration since it is established by the equivalence of charge conjugation and isoduality (Lemma 2.3.4), and we can write:

LEMMA 2.3.5 [6,5b,18], [7]: *All experimental data currently available for antiparticles represented via charge conjugation are equally verified by the isodual theory of antimatter.*

2.3.9 Elementary Particles and their Isoduals

We assume the reader is familiar with the conventional definition of *elementary particles* as irreducible unitary representations of the spinorial covering of the Galilei symmetry $G(3.1)$ for nonrelativistic treatments and those of the Poincaré symmetry $P(3.1)$ for relativistic treatments. We therefore introduce the following:

DEFINITION 2.3.1: Elementary isodual particles (antiparticles) are given by irreducible unitary representations of the spinorial covering of the Galilei-Santilli's isodual symmetry $G^d(3.1)$ *for nonrelativistic treatments and those of the Poincaré-Santilli isodual symmetry* $P^d(3.1)$ *for relativistic treatments.*

A few comments are now in order. Firstly, one should be aware that "isodual particles" and "antiparticles" do not represent the same notion, evidently because of the negative mass, energy and time of the former compared to positive mass, energy and time of the latter. In the rest of this chapter, unless otherwise stated, the word "antiparticle" will be referred to as the "isodual particle."

For instance the word "positron" e^+ is more appropriately intended to represent the "isodual electron" with symbol e^{-d}. Similarly the, "antiproton" p^- is intended to represent the "isodual proton" p^{+d}.

Secondly, the reader should note the insistence on the *elementary* character of the antiparticles here admitted. The reason is that the antigravity studied in Chapter 4 is specifically formulated for "elementary" isodual particles, such as the isodual electron, due to a number of unsettled aspects pertaining to composite particles.

Consider, as an illustration, the case of mesons. If the π° is a bound state of a particle and its isodual, the state is isoselfdual and, as such, it *cannot* experience antigravity, as illustrated in the next section. A number of ambiguities then follow for the study of the gravity of the charged mesons $\pi^\pm$, such as the problem of ascertaining which of the two mesons is a particle and which is its isodual or, whether the selected antiparticle is indeed the isodual image of the particle as a necessary condition for meaningful study of their gravity.

Note that essentially the same ambiguities prohibit the use of muons for a serious theoretical and experimental studies of the gravity of antiparticles, again, because of unsettled problems pertaining to the structure of the muons themselves. Since the muons are naturally unstable, they cannot be credibly believed to be elementary. Therefore, serious theoretical and experimental studies on the gravity of muons require the prior identification of their constituents with physical particles.

Finally, the reader should be aware that *Definition 2.3.1 excludes the use of quark conjectures for the gravitational studies of this monograph.* This is due to the well-known basic inconsistency of quark conjecture of not admitting any gravitation at all (see, e.g., the Appendix of Ref. [18]). In fact, gravity can only be defined in our spacetime while quarks can only be defined in their mathematical unitary internal space with no known connection with our spacetime due to the O'Rafearthaigh theorem.[5]

Also, the only "masses" that can be credibly claimed as possessing inertia are the eigenvalues of the second-order Casimir invariant of the Poincaré symmetry $p_\mu \times p^\mu = m^2$. Quarks cannot be characterized via such a fundamental symmetry, as well known. It then follows that "quark masses" are mere mathematical parameters defined in the mathematical internal complex-unitary space that cannot possibly be used as serious basis for gravitational tests.

2.3.10 Photons and their Isoduals

As it is well known, photons have no charge and, therefore, they are invariant under charge conjugation, as transparent from the simple plane-wave representation

$$\Psi(t,r) = N \times e^{i\times(k\times r - E\times t)}, \quad N \in R, \tag{2.3.59}$$

with familiar relativistic form

$$\Psi(x) = N \times e^{i\times k_\mu \times x^\mu}, \tag{2.3.60}$$

and familiar expression for the energy

$$E = h \times \nu. \tag{2.3.61}$$

As a result, matter and antimatter have been believed throughout the 20-th century to emit the same light. In turn, this belief has left fundamentally unsettled basic questions in astrophysics and cosmology, such as the lack of quantitative

[5]The possible connection between internal and spacetime symmetries offered by supersymmetric theories cannot be credibly used for gravitational tests due to their highly unsettled character and the prediction of a zoo of new particles none of which has been experimentally detected to the author's best knowledge.

studies as to whether far-away galaxies and quasars are made up of matter or of antimatter.

One of the most intriguing and far reaching implications of the isodual theory is that, while remaining evidently invariant under charge conjugation, *the photon is not invariant under isoduality,* thus admitting a conjugate particle first submitted by Santilli in Ref. [18] under the name of *isodual photon.* In particular, the isodual photon emerges as having physical characteristics that can be experimentally measured as being different from those of the photon.

Therefore, the isodual theory offers the first known possibilities of quantitative theoretical and experimental studies as to whether a far-away galaxy or quasar is made of matter or of antimatter due to detectable physical differences of their emitted light.

Note that the term "antiphoton" could be misleading because the prefix "anti" is generally assumed as referring to charge conjugation. For this reason the name of "isodual photon" appears to be preferable, also because it represents, more technically, the intended state.

In fact, the photon is mapped by isoduality into a new particle possessing all negative-definite physical characteristics, with the following simple isodual plane-wave representation

$$\Psi^d(t^d, r^d) = N^d \times^d e_d^{i^d \times^d (k^d \times^d r^d - E^d \times^d t^d)}, \;\; N^d \in R^d, \tag{2.3.62}$$

with relativistic expression on isodual Minkowski space

$$\Psi^d(x^d) = N^d \times^d e_d^{i^d \times^d k^d_\mu \times^d x^{d\mu}}, \tag{2.3.63}$$

and isodual expression for the energy

$$E^d \; = \; h^d \times^d \nu^d, \tag{2.3.64}$$

where e_d is the isodual exponentiation (2.1.26b).

Note that, since i is isoselfdual, Eq. (2.1.20), *the exponent of the plane-wave representation is invariant under both charge conjugation and isoduality,* as illustrated by the following expression

$$i^d \times^d (k^d \times^d r^d - E^d \times^d t^d) \; \equiv \; i \times (k \times r - E \times t), \tag{2.3.65}$$

or its relativistic counterpart

$$i^d \times^d k^d_\mu \times^d x^{d\mu} \; \equiv \; i \times k_\mu \times x^\mu, \tag{2.3.66}$$

thus confirming the lack of contradiction between charge conjugation and isoduality.

Moreover, both the photon and the isodual photon travel in vacuum with the same (absolute) speed $|c|$, for which we have the additional identity

$$k^d_\mu \times^d k^{d\mu} \equiv k_\mu \times k^\mu = 0. \tag{2.3.67}$$

Despite the above identities, energy and time are positive-definite for the photon, while they are negative-definite for the isodual photon. As we shall see, the latter property implies that photons are attracted by the gravitational field of matter while isodual photons are repelled, thus providing a physically detectable difference.

Additional differences between light emitted by matter and that emitted by antimatter, such as those pertaining to parity and other discrete symmetries, require additional study.

All in all, the isodual theory of antimatter permits the first possibilities known to the author for future experimental measurements as to whether far-away galaxies and quasars are made up of matter or of antimatter.

2.3.11 Electrons and their Isoduals

The next truly elementary particles and antiparticles are the electron e^- and its antiparticle, the positron e^+ or the isodual electron e^{-d}. The differences between the "positron" and the "isodual electron" should be kept in mind. In fact, the former has positive rest energy and moves forward in time, while the latter has negative rest energy and moves backward in time.

Also, the electron is known to experience gravitational attraction in the field of matter, as experimentally established. As conventionally defined, the positron too is predicted to experience gravitational attraction in the field of matter (because its energy is positive).

However, as we shall see in Chapter 4, the isodual electron is predicted to experience antigravity when immersed in the field of matter, and this illustrates again the rather profound physical differences between the "positron" and the "isodual electron".

Note that, in view of their truly elementary character, isodual electrons are the ideal candidates for the measurement of the gravitational field of antiparticles.

2.3.12 Protons and their Isoduals

The next particles demanding comments are the proton p^+, the antiproton p^- and the isodual proton p^{+d}. In this case the differences between the "antiproton" and the "isodual proton" should be kept in mind to avoid major inconsistencies with the isodual theory, such as the study of the possible antigravity for antiprotons in the field of matter which antigravity cannot exist for the isodual theory (due, again, to the positive mass of the antiproton).

Note that these particles are not elementary and, as such, they are not admitted by Definition 2.3.1. moreover, as stressed earlier [18], when represented in term of quark conjectures both the proton and the antiproton cannot admit any gravity at all, let alone antigravity. As a result, extreme scientific care should be exercised before extending to all antimatter any possible gravitational measurements for antiprotons.

2.3.13 The Hydrogen Atom and its Isodual

The understanding of this chapter requires the knowledge that studies conducted on the *antihydrogen atom* (see, e.g., the various contributions in Proceedings [19]), even though evidently interesting per se, have no connection with the *isodual hydrogen atom,* because the antihydrogen atom has positive mass, for which antigravity is prohibited, and emits conventional photons. Therefore, it is important to inspect the differences between these two formulations of the simplest possible atom of antimatter.

We assume as exactly valid the conventional quantum mechanical theory of *bound states of point-like particles at large mutual distances,*[6] as available in quantum mechanical books so numerous to discourage even a partial listing.

For the case of two particles denoted with the indices 1, 2, the total state in the Hilbert space is the familiar tensorial product of the two states

$$|\psi >= |\psi_1 > \times |\psi_2 > . \tag{2.3.68}$$

The total Hamiltonian H is the sum of the kinetic terms of each state plus the familiar interaction term $V(r)$ depending on the mutual distance r,

$$H = p_1 \times p_1/2 \times m_1 + p_2 \times p_2/2 \times m_2 + V(r). \tag{2.3.69}$$

The total angular momentum is computed via the familiar expressions for angular momenta and spins

$$J = J_1 \times I + I \times J_2, \ \ S = S_1 \times I + I \times S_2, \tag{2.3.70}$$

where the I's are trivial units, with the usual rules for couplings, addition, etc. One should note that the unit for angular momenta is three-dimensional while that for spin has a generally different dimension.

A typical example of two-body bound states of particles is the *hydrogen atom* that experiences attraction in the gravitational field of matter with the well established emission of conventional photons.

[6] We are here referring to the large mutual distances as occurring in the atomic structure and exclude the short mutual distances as occurring in the structure of hadrons, nuclei and stars since a serious study of the latter is dramatically beyond the capabilities of quantum mechanics, as shown beyond scientific doubt in Chapter 3.

The study of *bound states of point-like isodual particles at large mutual distances* is an important part of isodual quantum mechanics. These bound states can be studied via an elementary isoduality of the corresponding bound states for particles, that is, via the use of the isodual Hilbert spaces $\mathcal{H}^d$ studied earlier.

The *total isodual state* is the tensorial product of the two isodual states

$$|\psi^d(r^d) >^d = |\psi_1^d(r^d) >^d \times^d |\psi_2^d(r^d) >^d = - < \psi_1(-r)| \times < \psi_2(-r)|. \qquad (2.3.71)$$

The *total isodual Hamiltonian* is the sum of the isodual kinetic terms of each particle plus the isodual interaction term depending on the isodual mutual distance,

$$H^d = p_1^d \times^d p_1^d /^d 2^d \times^d m_1^d + p_2^d \times^d p_2^d /^d 2^d \times^d m_2^d + V^d(r^d). \qquad (2.3.72)$$

The total *isodual angular momentum* is based on the expressions for isodual angular momenta and spin

$$J^d = J_1^d \times^d I^d + I^d \times^d J_2^d, \qquad (2.3.73a)$$

$$S^d = S_1^d \times^d I^d + I^d \times^d S_2^d, \qquad (2.3.73b)$$

The remaining aspects (couplings, addition theory of angular momenta, etc.) are then given by a simple isoduality of the conventional theory that is here omitted for brevity.

Note that all eigenvalues that are positive for the conventional case measured with positive units become negative under isoduality, yet measured with negative units, thus achieving full equivalence between particle and antiparticle bound states.

The simplest possible application of the above isodual theory is that for the *isodual hydrogen atom* (first worked out in Ref. [18]). The novel predictions of isoduality over that of the antihydrogen atom is that the isodual hydrogen atom is predicted to experience antigravity in the field of matter and emits isodual photons that are also repelled by the gravitational field of matter.

2.3.14 Isoselfdual Bound States

Some of the most interesting and novel bound states predicted by the isodual theory are the *isoselfdual bound states,* that is, bound states that coincide with their isodual image. The simplest case is the bound state of one elementary particle and its isodual, such as the *positronium.*

The condition of isoselfduality requires that the basic symmetry must be itself isoselfdual, e.g., for the nonrelativistic case the total symmetry must be

$$G_{Tot} = G(3.1) \times G^d(3.1), \qquad (2.3.74)$$

where $\times$ is the Kronecker product (a composition of states thus being isoselfdual), with a simple relativistic extension here assumed as known from the preceding sections.

The total unit must also be isoselfdual,

$$I_{Tot} = I \times I^d, \tag{2.3.75}$$

where I represents the space, time and spin units.

The total Hilbert space and related states must also be isoselfdual,

$$\mathcal{H}_{Tot} = \mathcal{H} \times \mathcal{H}^d, \tag{2.3.76a}$$

$$|\psi>_{Tot} = |\psi> + |\psi>^d = |\psi> - <\psi|, \tag{2.3.76b}$$

and so on.

A main feature is that isoselfdual states exist in both the spacetime of particles and that of antiparticles. Therefore, the computation of the total energy must be done *either* in $\mathcal{H}$, in which case the total energy is positive, *or* in $\mathcal{H}^d$, in which case the total energy is negative.

Suppose that a system of one elementary particle and its isodual is studied in our laboratory of matter. In this case the eigenvalues for both particle and its isodual must be computed in $\mathcal{H}$, in which case we have the equation

$$\begin{aligned} i \times \partial_t |\psi> &= (p \times p/2 \times m) \times |\psi> + \\ +(p^d \times^d p^d/^d 2^d \times^d m^d) \times^d |\psi> &+ V(r) \times |\psi> = \\ = \ [p \times p/2 \times m + V(r)] &\times |\psi> = E \times |\psi>, \end{aligned} \tag{2.3.77}$$

under which the total energy E is evidently positive.

When the same isoselfdual state is detected in the spacetime of antimatter, it must be computed with respect to $\mathcal{H}^d$, in which case the total energy is negative, as the reader is encouraged to verify.

The total angular momentum and other physical characteristics are computed along similar lines and they also result in having positive values when computed in $\mathcal{H}$, as occurring for the conventional charge conjugation.

As we shall see shortly, the positive character of the total energy of bound states of particles and their antiparticles is crucial for the removal of the inconsistencies of theories with negative energy.

The above properties of the isoselfdual bound states have the following implications:

1) Isoselfdual bound states of elementary particles and their isoduals are predicted to be attracted in both, the gravitational field of matter and that of antimatter because their total energy is positive in our world and negative in the isodual world. This renders necessary an experimental verification of the gravitational behavior of isoselfdual bound states, independently from that of individual

antiparticles. Note that the prediction holds only for bound states of truly elementary particles and their isoduals, such as the positronium. No theoretical prediction for the muonium and the pionium is today feasible because the unsettled nature of their constituents.

2) Isoselfdual bound states are predicted to have a null internal total time $t + t^d = 0$ and therefore acquires the time of the matter or antimatter in which they are immersed, although the physical time t of the observer (i.e., of the bound state equation) is not null. This is readily understood by noting that the quantity t of Eq. (2.3.77) is our own time, i.e., we merely study the behavior of the state with respect to our own time. A clear understanding illustrated previously with the "isodual cube" of Section 2.1 is that the description of a state with our own time, by no means, implies that its intrinsic time necessarily coincides with our own. Note that a similar situation occurs for the energy because the intrinsic total energy of the positronium is identically null, $E + E^d = 0$. Yet the energy measured by us is $E_{part.} - E^d_{antipart.} = 2E > 0$. A similar situation occurs for all other physical quantities.

3) Isoselfdual bound states may result in being the microscopic image of the main characteristics of the entire universe. Isoselfduality has in fact stimulated a new cosmology, the *isoselfdual cosmology* [21] studied in Chapter 5, that is patterned precisely along the structure of the positronium or of Dirac's equation in our isoselfdual re-interpretation. In this case the universe results in having null total physical characteristics, such as null total energy, null total time, etc., thus implying no discontinuity at its creation.

2.3.15 Resolution of the Inconsistencies of Negative Energies

The treatment of antiparticles with negative energies was rejected by Dirac because of incompatibility with their physical behavior. Despite several attempts made during the 20-th century, the inconsistencies either directly or indirectly connected to negative energies have remained unresolved.

The isodual theory of antimatter resolves these inconsistencies for the reason now familiar, namely, that the inconsistencies emerge when one refers negative energies to conventional numbers with positive units, while the same inconsistencies cannot be evenly formulated when negative energies are referred to isodual numbers and their negative units.

A good illustration is given by the known objection according to which the creation of a photon from the annihilation of an electron-positron pair, with the electron having a positive energy and the positron having a negative energy, would violate the principle of conservation of the energy.

In fact, such a pair could be moved upward in our gravitational field without work and then annihilated in their new upward position. The resulting photon would then have a blueshift in our gravitational field of Earth, thus having more energy than that of the original photon.

Presumed inconsistencies of the above type cannot be even formulated within the context of the isodual theory of antimatter because, as shown in the preceding section, the electron-positron state is isoselfdual, thus having a non-null *positive* energy when observed in our spacetime. Consequently, the lifting upward of the pair does indeed require work and no violation of the principle of conservation of the energy can be expected.

A considerable search has established that all other presumed inconsistencies of negative energy known to the author cannot even be formulated within the context of the isodual theory of antimatter. Nevertheless, the author would be particularly grateful to any colleague who brings to his attention inconsistencies of negative energies that are really applicable under negative units.

2.4 THEORETICAL PREDICTIONS OF ANTIGRAVITY

2.4.1 The Problem for Studies on Antigravity: Ethical Decay in Physics

Antigravity is one of the most ancient dreams of mankind, that has stimulated the imagination of many researchers, from various engineering fields (see, e.g., Refs. [35, 36] that also list patents), to the most advanced branches of physics (see the prediction of antigravity in supergravity theories [37, 38] and proceedings [19] for other more recent approaches).

Unfortunately, professional theoretical and experimental research in antigravity has been opposed, disrupted, or jeopardized by organized academic, financial and ethnic interests on Einsteinian doctrines on grounds that antigravity is not predicted by said doctrines. However, as we known to experts in order to qualify as such, and as established beyond credible doubt in this volume, Einsteinian doctrines do not allow a consistent classical representation of antimatter.

Hence, any opposition, obstruction, of dismissal of antigravity based on Einsteinian doctrines is sheer scientific corruption for personal gains that must be denounced by any scientist who really cares about scientific knowledge and human dignity.

A comprehensive study of antigravity was conducted by the author in monograph [34]. In this chapter we essentially present an update of the content of Ref. [34].

An experiment on the gravity of antiparticles was considered by Fairbank and Witteborn [39] via low energy positrons in vertical motion. Unfortunately, the measurements could not be completed because of claimed interferences from stray fields, excessive upward kinetic energy of the positrons, and other reasons.

There are insistent rumors that the experiment by Fairbank and Witteborn could not be completed because of disruptions by said organized interests at SLAC and elsewhere, particularly in view of a growing expectation that the experiment could indeed establish antigravity between matter and antimatter, thus cutting out of the desired dominance by Einsteinian doctrines what could amount as being half of the universe,

. The author recommends a senatorial investigation of the case to ascertain the reasons for the lack of completion at SLAC of so important an experiment, in view of its feasibility with available technologies, as well as moderate cost, while dramatically more expensive experiments fully aligned with Einsteinian doctrines were preferred at SLAC and elsewhere, and remain preferred to this day (January 19, 2008).

In the absence of a senate investigation, the author recommends the filing of a class action in the U. S. Federal Court against SLAC on grounds of misuses of public funds and other easily identifiable violations of Federal laws.

As an illustration of the need for a senatorial investigation and/or legal action, the reader in good faith should know that Burton Richter, then SLAC director, prohibited in writing in the early 1990s Santilli (a U.S. scientist) to visit SLAC (a U. S. federal laboratory) for the purpose of discussing a possible alternative of the Fairbank and Witteborn experiment via a horizontal tube (see Section 2.5.2 and references quoted therein), even though Santilli had applied for a visit fully supported by his own money and, being a theoretician, was merely looking for suggestions by experimental colleagues.

Clearly, such a denial by Burton Richter cannot be justified on scientific grounds, or on grounds of qualifications, since SLAC is bound by law as being open to visits from a variety of scientists, and Santilli qualification, honors and publications surpass most of the visitors (see Santilli CV http://www.i-b-r.org/-Ruggero-Maria-Santilli.htm www.santilli-galilei.com www.i-b-r.org and other sources).

Hence, the denial by Burton Richter for Santilli to visit SLAC, pushed to the extreme of perpetrating a clear violation of U. S. Federal Laws governing federal laboratories, had strictly nonscientific motivations. The most plausible one is the evident one, namely, the existence at SLAC and elsewhere of vociferous, organized, academic, financial and ethnic interests on Einsteinian doctrines opposing, disrupting and jeopardizing for asocial personal gains professional research on antigravity. At any rate, if the above, and much more evidence by other scientists, is not sufficient for a senatorial investigation and/or legal action, what else would be?

In view of the above, any consideration of experiments on the gravity between particle and antiparticles without a senatorial and/or judicial investigation of the past, would be a hypocritical farce acceptable by naive persons or accomplices, but definitely not in favor of scientific knowledge. At any rate, in the event said organized academic, financial and ethnic interests on Einstein do not exist, a senatorial and/or judicial process would indeed establish their lack of existence. So, why oppose effective ways for their dismissal?

2.4.2 Outline of the Literature on Antigravity

Besides the above quoted Refs. [35, 36], [37, 38], [39], additional data on the gravity of antiparticles are those from the LEAR machine on antiprotons at CERN [40], although these data too are inconclusive because of the excessive energy of the antiprotons and other factors, including the care necessary to extend the gravity of antiprotons to all antiparticles pointed out in Chapter 2, the proved impossibility for quarks to experience gravity, let alone antigravity, and other factors.

Additional experiments on the gravity of antiparticles are based on neutron interferometry, such as the experiments by Testera [41], Poggiani [42] and others. These experiments are extremely sensitive and, as such, definite and conclusive results continue to be elusive. In particular, the latter experiments too deal with antiprotons, thus inheriting the ambiguities of quark conjectures with respect to gravity, problems in the extension to other antiparticles, and other open issues.

All further data on the gravity of antiparticles known to this author are of indirect nature, e.g., via arguments based the equivalence principle (see, e.g., Ref. [33] and papers quoted therein). Note that the latter arguments do not apply under isoduality and will not be considered further.

A review on the status of our knowledge prior to isodual theories is available in Ref. [43], that includes an outline of the arguments against antigravity, such as those by Morrison, Schiff and Good. As we shall see, the latter arguments too cannon even be formulated under isodualities, let alone be valid.

We can therefore conclude by stating that at this writing there exists no experimental or theoretical evidence known to this author that is resolutory and conclusive either against or in favor of antigravity.

One of the most intriguing predictions of isoduality is the existence of *antigravity* conceived as a reversal of the gravitational attraction, first theoretically submitted by Santilli in Ref. [44] of 1994.

The proposal consists of an experiment that is feasible with current technologies and permits a definite and final resolution on the existence or lack of the existence of the above defined antigravity.

These goals were achieved by proposing the test of the gravity of positrons in horizontal flight on a vacuum tube. The experiment is resolutory because, for the case of a 10 m long tube and very low kinetic energy of the positrons (of the order of μeV), the displacement of the positrons due to gravity is sufficiently large to be visible on a scintillator to the naked eye.

Santilli's proposal [44] was studied by the experimentalist Mills [45] to be indeed feasible with current technology, resolutory and conclusive.

The reader should be aware from these introductory lines that *the prediction of antigravity exists, specifically, for the isodual theory of antimatter and not for conventional treatment of antiparticles.*

For instance, no prediction of antigravity can be obtained from Dirac's hole theory or, more generally, for the treatment of antimatter prior to isoduality, that solely occurring in second quantization.

Consequently, antigravity can safely stated to be the ultimate test of the isodual theory of antimatter.

In this chapter, we study the prediction of antigravity under various profiles, we review the proposed resolutory experiment, and we outline some of the far reaching implications that would follow from the possible experimental verification of antigravity, such as the consequential existence of a fully *Causal Time Machine,* although not for ordinary matter, but for an isoselfdual combination of matter and antimatter.

2.4.3 Newtonian and Euclidean Prediction of Antigravity

It is important to show that the prediction of antigravity can be first formulated at the most primitive possible level, that of Newtonian mechanics and its isodual. All subsequent formulations will be merely consequential.

The current theoretical scene on antigravity is dominated by the fact that, as it is well known, the Euclidean, Minkowskian and Riemannian geometries offer no realistic possibility to reverse the sign of a gravitational mass or of the energy of the gravitational field.

Under these conditions, existing theories can at best predict a decrease of the gravitational force of antiparticles in the field of matter (see Ref. [43] for a review of these conventional studies). In any case the decreased interaction, as such, remains attractive.

Isodual mathematical and physical theories alter this scientific scene. In fact, antigravity is predicted by the interplay between the classical Euclidean geometry and its isodual. The resulting prediction of antigravity persists at all levels, that is, for flat and curved spaces and for classical or quantum formulations, in a fully consistent way without known internal contradictions.

Also, antigravity is a simple consequence of Corollary 2.3.1 according to which the observed trajectories of antiparticles under a magnetic field are the *projection* in our spacetime of inverted trajectories in isodual spacetime.

Once these aspects are understood, the prediction of antigravity becomes so simple to appear trivial. In fact, antigravity merely originates from the projection of the gravitational field of matter in that of antimatter and vice-versa. We therefore have the following:

PREDICTION 2.4.1 [43, 5]: *The existence of antigravity, defined as a gravitational repulsion experienced by isodual elementary particles in the field of matter and vice-versa, is a necessary consequence of a consistent classical description of antimatter.*

Let us begin by studying this prediction in Euclidean and isodual Euclidean spaces. Consider the Newtonian gravitational force of two conventional (thus, positive) masses m_1 and m_2

$$F = -G \times m_1 \times m_2 / r < 0, \quad G, m_1, m_2 > 0, \tag{2.4.1}$$

where G is the gravitational constant and the minus sign has been used for similarity with the Coulomb law.

Within the context of conventional theories, the masses m_1 and m_2 remain positive irrespective of whether referred to a particle or an antiparticle. This yields the well known "universal law of Newtonian attraction", namely, the prediction that the gravitational force is attractive irrespective of whether for particle-particle, antiparticle-antiparticle or particle-antiparticle.

Again, the origin of this prediction rests in the assumption that antiparticles exist in our spacetime, thus having positive masses, energy and time. Under isoduality the situation is different. For the case of antiparticle-antiparticle under isoduality we have the different law

$$F^d = -G^d \times^d m_1^d \times^d m_2^d /^d r^d > 0, \quad G^d, m_1^d, m_2^d < 0. \tag{2.4.2}$$

But this force exists in the different isodual space and is defined with respect to the negative unit -1. Therefore, isoduality correctly represents the *attractive* character of the gravitational force between two isodual particles.

The case of particle-antiparticle under isoduality requires the *projection* of the isodual particle in the space of the particle (or vice versa), and we have the law

$$F = -G \times m_1 \times m_2^d / r > 0, \tag{2.4.3}$$

that now represents a *repulsion,* because it exists in our spacetime with unit $+1$, and it is opposite to force (2.4.1). This illustrates antigravity as per Prediction 2.4.1 when treated at the primitive Newtonian level.

Similarly, if we project the particle in the spacetime of the antiparticle, we have the different law

$$F^d = -G^d \times^d m_1^d \times^d m_2 /^d r^d < 0, \tag{2.4.4}$$

that also represents *repulsion* because referred to the unit -1.

We can summarize the above results by saying that *the classical representation of antiparticles via isoduality renders gravitational interactions equivalent to the electromagnetic ones, in the sense that the Newtonian gravitational law becomes equivalent to the Coulomb law, thus necessarily including both attraction and repulsions.*

The restriction in Prediction 2.4.1 to "elementary" isodual particles will soon turn out to be crucial in separating science from its political conduct, and *de facto* restricts the experimental verification of antigravity to positrons in the field of Earth.

Note also that Prediction 2.4.1 is formulated for "isodual particles" and *not* for antiparticles. This is due to the fact indicated in preceding sections that, according to current terminologies, antiparticles are defined in our spacetime and have positive masses, energy and time. As such, no antigravity of any type is possible for antiparticles as conventionally understood.

2.4.4 Minkowskian and Riemannian Predictions of Antigravity

It is important to verify the above prediction at the classical relativistic and gravitational levels.

Let $M(x, \eta, R)$ be the conventional Minkowskian spacetime with coordinates $x = (r, t)$ (as a column) and metric $\eta = Diag.(1, 1, 1, -1)$ over the field of real numbers $R(n, +, \times)$ with unit $I = Diag.(1, 1, 1, 1)$. The *Minkowski-Santilli isodual space* [8] is given by (Section 2.2.8)

$$M^d(x^d, \eta^d, R^d), \quad x^d = -x^t, \quad \eta^d = Diag.(-1, -1, -1, +1), \tag{2.4.5a}$$

$$I^d = Diag.(-1, -1, -1, -1). \tag{2.4.5b}$$

The *isodual electromagnetic field* on $M^d(x^d, \eta^d, R^d)$ is given by

$$F^d_{\mu\nu} = \partial^d_\nu A^d_\mu - \partial^d_\mu A^d_\nu = -F^d_{\nu\mu}, \quad \mu, \nu = 1, 2, 3, 4, \tag{2.4.6}$$

with *isodual energy-momentum tensor*

$$T^d_{\mu\nu} = (1^d/^d 4^d \times m^d) \times^d [F^{d\,\alpha}_\mu \times^d F^d_{\alpha\nu} +$$

$$+ (1^d/^d 4^d) \times^d g^d \times^d F^d_{\alpha\beta} \times^d F^{d\,\alpha\beta}] = -T^t_{\nu\mu}, \tag{2.4.7}$$

where g is a known constant depending on the selected unit (whose explicit value is irrelevant for this study). Most importantly, the fourth component of the isodual energy-momentum tensor is negative-definite,

$$T^d_{00} < 0. \tag{2.4.8}$$

As such, antimatter represented in isodual Minkowski geometry has negative-definite energy, and other physical characteristics, and evolves backward in time. It is an instructive exercise for the interested reader to prove that the results of the Newtonian analysis of the preceding section carry over in their entirety to the Minkowskian formulation [8].

Consider now a Riemannian space $\mathcal{R}(x, g, R)$ in (3+1)-dimensions with spacetime coordinates x and metric $g(x)$ over the reals R with basic unit $I = Diag.(1, 1, 1, 1)$ and related Riemannian geometry as presented, e.g., in Refs. [33, 47]. As outlined in Section 2.1.7, the *isodual iso-Riemannian spaces* are given by

$$\mathcal{R}^d(x^d, g^d, R^d) : x^d = -x^t, \;\; g^d(x^d) = -g^t(-x^t), \tag{2.4.9a}$$

$$I^d = Diag.(-1, -1, -1, -1). \tag{2.4.9b}$$

Recall that a basic drawback in the use of the Riemannian geometry for the representation of antiparticles is the positive-definite character of its energy-momentum tensor.

In fact, this character causes unsolved inconsistencies at all subsequent levels of study of antimatter, such as lack of a consistent quantum image of antiparticles.

These inconsistencies are resolved *ab initio* under isoduality. In fact, the isodual Riemannian geometry is defined over the isodual field of real numbers R^d for which the norm is negative-definite (Section 2.2.1).

As a result, all quantities that are positive in Riemannian geometry become negative under isoduality, thus including the energy-momentum tensor. In particular, energy-momentum tensors in the Riemannian geometry are given by relativistic expression (2.1.49i) and, as such, they remain negative-definite when treated in a Riemannian space.

It then follows that in the isodual Riemannian treatment of the gravity of antimatter, all masses and other quantities are negative-definite, including the *isodual curvature tensor,* Eq. (2.1.49c).

Despite that, the gravitational force between antimatter and antimatter remain *attractive,* because said negative curvature is measured with a negative unit.

As it was the case at the preceding Euclidean and Minkowskian levels, the isodual treatment of the gravitation of matter-antimatter systems requires its projection *either* in our spacetime *or* in the isodual spacetime. This again implies a *negative curvature in our spacetime* [8] resulting in Prediction 2.4.1 of antigravity at the classical Riemannian level too.

2.4.5 Prediction of Antigravity from Isodual Einstein's Gravitation

Einstein's gravitation is generally defined (see, e.g., Ref. [33]) as the reduction of gravitation in the exterior problem in vacuum to pure curvature in a Riemannian space $\mathcal{R}(x, g, R)$ with local spacetime coordinates x and metric $g(x)$ over the field of real numbers R *without a source,* according to the celebrated field equations

$$G_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu} \times R/2 = 0, \tag{2.4.10}$$

where $G_{\mu\nu}$ is generally referred to as the *Einstein tensor,* $R_{\mu\nu}$ is the *Ricci tensor,* and R is the *Ricci scalar.*

As it is well known, *Einstein's conception of gravitation as above identified does not permit antigravity,* and this occurrence has been a motivation for the absence of serious experimental studies in the field, as indicated in Section 1.4.1.

However, we have indicated in preceding chapters that *the problem of antigravity cannot be confidently formulated, let alone treated, in Einstein's gravitation, due to the impossibility of consistently treating antimatter.*

As indicated earlier, the only possible formulation of antimatter is that by *only* changing the sign of the charge. However, this formulation is inconsistent with quantization since it leads to particles, rather than antiparticles, with the wrong sign of the charge.

At any rate, *the most important formulation of the gravity of antimatter is that for astrophysical bodies with null total charge, as expected for an antimatter star or an antimatter neutron star.*

The impossibility for any credible treatment of antimatter is then established by the fact that *according to Einstein's conception of gravitation the gravitational fields equations for matter and antimatter stars with null total charge are identical.*

These inconsistencies are resolved by the isodual theory of antimatter because it implies the novel *isodual field equations for antimatter* defined on the isodual Riemannian space [8] $\mathcal{R}^d(x^d, g^d, R^d)$ with local isodual spacetime coordinates $x^d = -x^t$ and isodual metric $g^d(x^d) = -g^t(-x^t)$ over the isodual field of real numbers R^d

$$G^d_{\mu\nu} = R^d_{\mu\nu} - g^d_{\mu\nu} \times R^d/^d 2^d = 0. \tag{2.4.11}$$

The latter representation is based on a negative-definite energy-momentum tensor, thus having a consistent operator image, as shown in Chapter 3.

We, therefore, conclude this analysis with the following:

THEOREM 2.4.1 : Antigravity is a necessary and sufficient condition for the existence of a classical formulation of antimatter compatible with its operator counterpart.

Proof. Assume the validity of Einstein's gravitation for matter and its isodual for antimatter. Then, the former has a positive curvature tensor and the latter has a negative curvature tensor.

Therefore, the projection of the gravitational field of antimatter in the spacetime of matter implies a negative curvature tensor in our spacetime, namely, antigravity, or, vice-versa, a positive curvature tensor in the isodual spacetime, that is also repulsive, and this proves the sufficiency. The necessity comes from the fact that the only formulation of antimatter compatible with operator counterparts is that based on negative energies and masses.

In turn, geometric formulations of negative energies and masses necessarily imply, for consistency, a negative curvature tensor. Still in turn, when projected in the space of matter, a negative curvature necessarily implies antigravity and the same occurs for the projection of matter in the field of antimatter. **q.e.d.**

2.4.6 Identification of Gravitation and Electromagnetism

In addition to the above structural inability by Einstein's equations (2.4.10) to represent antimatter, Einstein's gravitation is afflicted by a litany of inconsistencies for the treatment of *matter itself* studied in Section 1.4 whose resolution requires a number of structural revisions of general relativity.

It is important to show that the prediction of antigravity, not only persists, but it is actually reinforced for gravitational theories resolving the inconsistencies of Einstein's gravitation.

The first catastrophic inconsistency of Einstein's gravitation crucial for the problem of antigravity is that of Theorem 1.4.1 on the irreconcilable incompatibility between Einstein's lack of source in vacuum and the electromagnetic origin of mass.

As stressed in Section 1.4, this inconsistency is such that, either one assumes Einstein's gravitation as correct, in which case quantum electrodynamics must be reformulated from its foundation to prevent a first-order source in vacuum, or one assumes quantum electrodynamics to be correct, in which case Einstein's gravitation must be irreconcilably abandoned.

The second catastrophic inconsistency of Einstein's gravitation is that of Theorem 1.4.2 identifying the incompatibility of field equations (2.4.10) and the forgotten Freud identity of the Riemannian geometry,

$$R^{\alpha}_{\beta} - \frac{1}{2} \times \delta^{\alpha}_{\beta} \times R - \frac{1}{2} \times \delta^{\alpha}_{\beta} \times \Theta \ = \ U^{\alpha}_{\beta} + \partial V^{\alpha\rho}_{\beta} / \partial x^{\rho} \ = \ k \times (t^{\alpha}_{\beta} + \tau^{\alpha}_{\beta}), \quad (2.4.12)$$

where

$$\Theta \ = \ g^{\alpha\beta} g^{\gamma\delta} (\Gamma_{\rho\alpha\beta} \Gamma^{\rho}_{\gamma\beta} - \Gamma_{\rho\alpha\beta} \Gamma^{\rho}_{\gamma\delta}), \quad (2.4.13a)$$

$$U^{\alpha}_{\beta} \ = \ -\frac{1}{2} \frac{\partial \Theta}{\partial g^{\rho\alpha}_{|\rho}} g^{\gamma\beta} \uparrow_{\gamma}, \quad (2.4.13b)$$

$$V^{\alpha\rho}_{\beta} \ = \ \frac{1}{2} [g^{\gamma\delta} (\delta^{\alpha}_{\beta} \Gamma^{\rho}_{\alpha\gamma\delta} - \delta^{\rho}_{\beta} \Gamma^{\rho}_{\alpha\delta}) +$$

$$+ (\delta^{\rho}_{\beta} g^{\alpha\gamma} - \delta^{\alpha}_{\beta} g^{\rho\gamma}) \Gamma^{\delta}_{\gamma\delta} + g^{\rho\gamma} \Gamma^{\alpha}_{\beta\gamma} - g^{\alpha\gamma} \Gamma^{\rho}_{\beta\gamma}]. \quad (2.4.13c)$$

The latter inconsistency requires the addition in the right-hand-side of Eqs. (2.4.10) of *two source tensors* for astrophysical bodies with null total charge.

As stressed in Section 1.4, the above two inconsistencies are deeply inter-related because complementary to each other, since the inconsistency of Theorem 1.4.2 is the dynamical counterpart of the inconsistency of Theorem 1.4.2 on geometric grounds.

A systematic study of the resolution of these inconsistencies was conducted by Santilli [48] in 1974.

The classical gravitational formulation of antimatter can be done in the *Riemannian-Santilli isodual space* $\mathcal{R}^d(x^d, g^d, R^d)$ studied in Sections 2.1.7 and 2.2.11.

To avoid catastrophic inconsistencies, the field equations of antimatter should be compatible with the basic geometric axioms of the isodual Riemannian geometry, including, most importantly, the *isodual Freud identity* [8], that can be written

$$R^{\alpha d}_{\beta} - \frac{1}{2}^d \times^d \delta^{\alpha d}_{\beta} \times^d R^d - \frac{1}{2}^d \times^d \delta^{\alpha d}_{\beta} \times^d \Theta^d = k^d \times^d (T^{d\alpha}_{\beta} + \Upsilon^{d\alpha}_{\beta}). \qquad (2.4.14)$$

with corresponding isodualities for Eqs. (2.4.13) here assumed as known.

These studies then leads to the following:

PREDICTION 2.4.2: [48] IDENTIFICATION OF GRAVITATION AND ELECTROMAGNETISM. In the exterior problem in vacuum, gravitation coincides with the electromagnetic interactions creating the gravitational mass with field equations

$$G^{Ext.}_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu} \times R/2 = k \times T^{Elm}_{\mu\nu}, \qquad (2.4.15)$$

where the source tensor $T^{Elm}_{\mu\nu}$ *represents the contribution of all charged elementary constituents of matter with resulting gravitational mass*

$$m^{Grav} = \int d^3x \times T^{Elm}_{00}, \qquad (2.4.16)$$

while in the interior problem gravitation coincides with electromagnetic interactions plus short range weak, strong and other interactions creating the inertial mass with field equations

$$G^{Int.}_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu} \times R/2 = k \times (T^{Elm}_{\mu\nu} + \Upsilon^{ShortRange}_{\mu\nu}), \qquad (2.4.17)$$

where the source tensor $\Upsilon^{ShortRange}_{\mu\nu}$ *represents all possible short range interactions in the structure of matter, with inertial mass*

$$m^{Inert} = \int d^3x \times (T^{Elm}_{00} + \Upsilon^{ShortRange}_{00}), \qquad (2.4.18)$$

and general law

$$m^{Inert} > m^{Grav}. \qquad (2.4.19)$$

The same identification of gravitation and electromagnetism then exists for antimatter with field equations and mass expressions given by a simple isodual form of the preceding ones.

A few comments are in order. All studies on the problem of "unification" of gravitation and electromagnetism prior to Ref. [48] known to this author[7] treated the two fields as *physically distinct*, resulting in the well known historical failures to achieve a consistent *unification* dating back to Albert Einstein (see next chapter for a detailed study). An axiomatically consistent theory emerges if gravitation and electromagnetism are instead "identified", as first done by Santilli [48] in 1974.

Also, Prediction 2.4.2 implies a *theory on the origin of the gravitational field,* rather than a theory providing its "description", as available in standard treatises such as [33]. This is due to the fact that in Prediction 2.4.2 *all* mass terms are completely eliminated and replaced with the fields originating mass.

In this way, the use of any mass term in any theory is an admission of our ignorance in the structure of the considered mass.

We should indicate for completeness that the identification of exterior gravitational and electromagnetic fields appears to be disproved by the assumption that quarks are physical constituents of hadrons, owing to the known large value of their "masses".

However, as indicated in Chapter 1, gravitation solely exists in our spacetime and cannot be consistently extended to mathematical unitary symmetries. Also, the only masses that can consistently create gravitation are those defined in our spacetime, thus necessarily being the eigenvalues of the second-order Casimir invariant of the Poincaré symmetry.

Since quarks cannot be defined in our spacetime, they cannot be consistently characterized by the Poincaré symmetry and their masses are not the eigenvalues of the second-order Casimir invariant of the latter symmetry, the use of quark masses has no scientific value in any gravitational profile. This is the reason why quark "masses" have been ignored in Ref. [48] as well as in this chapter.

It is well established in quantum electrodynamics that the mass of the electron is entirely of electromagnetic origin. Therefore, a gravitational theory of the electron in which the source term solely represents the charge contribution is incompatible with quantum electrodynamics. In fact, the latter requires *the entire reduction of the electron mass to electromagnetic fields* according to Eqs. (2.4.16).

[7] Again, the author would appreciate the indication of similar contributions prior to 1974.

Note in particular that, since the electron has a point-like charge, we have no distinction between exterior and interior problems with consequential identity

$$m_{Electron}^{Grav} \equiv m_{Electron}^{Inert}. \tag{2.4.20}$$

When considering a neutral, extended and composite particle such as the π°, the absence of a source tensor of electromagnetic nature renders gravitation, again, incompatible with quantum electrodynamics, as established in Ref. [48] and reviewed in Section 1.4.

By representing the π° as a bound state of a charged elementary particle and its antiparticle in high dynamical conditions, quantum electrodynamics establishes the existence not only of a non-null total electromagnetic tensor, but one of such a magnitude to account for the entire gravitational mass of the π° according to Eq. (2.4.16) and gravitational mass

$$m_{\pi^\circ}^{Grav} = \int d^3x \times T_{00\,\pi^\circ}^{Elm}. \tag{2.4.21}$$

Unlike the case of the electron, the π° particle has a very large charge distribution for particle standards. Moreover, the structure of the π° particle implies the additional weak and strong interactions, and their energy-momentum tensor is not traceless as it is the case for the electromagnetic energy-momentum tensor.

Therefore, for the case of the π° particle, we have a well-defined difference between exterior and interior gravitational problems, the latter characterized by Eqs. (2.4.18), i.e.,

$$m_{\pi^\circ}^{Inert} = \int d^3x \times (T_{00}^{Elm} + \Upsilon_{00}^{ShortRange}) \ > m_{\pi^\circ}^{Grav}. \tag{2.4.22}$$

The transition from the π° particle to a massive neutral star is conceptually and technically the same as that for the π°. In fact, the star itself is composed of a large number of elementary charged constituents each in highly dynamical conditions and, therefore, each implying a contribution to the total gravitational mass of the star as well as to its gravitational field.

The separation between exterior and interior problems, the presence of only one source tensor for the exterior problem and two source tensors for the interior problems, and the fact that the inertial mass is bigger than the gravitational mass is the same for both the π° and a star with null total charge.

For the case of a star we merely have an increased number of elementary charged constituents resulting in the expression [48]

$$m_{Star}^{Grav} = \Sigma_{p=1,2,3,\ldots} \int d^3x \times T_{00}^{Elem.Constit.}. \tag{2.4.23}$$

Note that when the star has a non-null total charge there is no need to change field equations (2.4.15) since the contribution from the total charge is automatically provided by the constituents.

As it is well known, there exist numerous other theories on the identity as well as the possible differentiation of gravitational and inertial masses (see, e.g., Ref. [33]). However, these theories deal with exterior gravitational problems while the studies here considered deal with the interior problem, by keeping in mind that inertial masses are a strictly *interior* problem, the exterior problem providing at best a geometric abstraction.

Nevertheless, one should remember that all these alternative theories are crucially based on Einstein's gravitation, while the theory presented in this section is based on quantum electrodynamics. Therefore, *none of the existing arguments on the differences between gravitational and inertial masses is applicable to the theory here considered.*

Note finally that conventional electromagnetism is represented by a *first-order tensor*, the electromagnetic tensor $F_{\mu\nu}$ of type (2.2.37a) and related first-order Maxwell's equations (2.2.37b) and (2.2.37c).

When electromagnetism is identified with exterior gravitation, it is represented with a *second-order tensor,* the energy-momentum tensor $T_{\mu\nu}$ of type (2.4.7) and related second-order field equations (2.4.15).

2.4.7 Prediction of Antigravity from the Identification of Gravitation and Electromagnetism

Another aspect important for this study is that *the identification of gravitation and electromagnetism in the exterior problem in vacuum implies the necessary existence of antigravity.*

In fact, the identification implies the necessary equivalence of the phenomenologies of gravitation and electromagnetism, both of them necessarily experiencing attraction and repulsion.

Note that this consequence is intrinsic in the identification of the two fields and does not depend on the order of the field equations (that is first order for electromagnetism and second order for gravitation as indicated earlier.

Alternatively, for the exterior problem of matter we have the field equations on $\mathcal{R}(x, g, R)$ over R

$$G^{Ext.}_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu} \times R/2 = k \times T^{Elm}_{\mu\nu}, \tag{2.4.24}$$

in which *the curvature tensor is positive,* and for the exterior problem of antimatter we have the isodual equations on $\mathcal{R}^d(x^d, g^d, R^d)$ over R^d

$$G^{d,Ext.}_{\mu\nu} = R^d_{\mu\nu} - g^d_{\mu\nu} \times R^d/2 = k \times T^{d,Elm}_{\mu\nu}, \tag{2.4.25}$$

in which *the curvature tensor is negative.*

The prediction of antigravity, Prediction 2.4.1, follows as a trivial extension of that of the preceding sections and occurs when the gravitational field of antimatter is projected in that of matter, or vice-versa, since such a projection implies a negative curvature in a Riemannian space that, by definition, is antigravity.

The prediction of antigravity is so strong that it is possible to prove that *the lack of existence of antigravity would imply the impossibility of identifying gravitation and electromagnetism.*

In turn, the lack of such identification would necessary require the impossibility for masses to have appreciable electromagnetic origin, resulting in the need for a structural revision of the entire particle physics of the 20-th century.

2.4.8 Prediction of Gravitational Repulsion for Isodual Light Emitted by Antimatter

Another important implication of the isodual theory of antimatter is the prediction that antimatter emits a new light, the *isodual light,* that experiences repulsion when in the vicinity of the gravitational field of matter, or vice-versa [18], where the *isodual electromagnetic waves* emitted by antimatter are given by Eqs. (2.3.37), i.e.,

$$F^d_{\mu\nu} \;=\; \partial^d A^d_\mu /^d \partial^d x^{\nu d} - \partial^d A^d_\nu /^d \partial^d x^{d\mu}, \tag{2.4.26a}$$

$$\partial^d_\lambda F^d_{\mu\nu} + \partial^d_\mu F^d_{\nu\lambda} + \partial^d_\nu F^d_{\lambda\mu} = 0, \tag{2.4.26b}$$

$$\partial^d_\mu F^{d\mu\nu} = -J^{d\nu}. \tag{2.4.26c}$$

The gravitational repulsion then emerges from the negative energy of the above isodual waves when in the field of matter. Vice versa, electromagnetic waves emitted by matter are predicted to experience antigravity when in the gravitational field of antimatter because they have a positive energy.

Note that *isodual electromagnetic waves coincide with conventional waves under all known interactions except gravitation.* Alternatively, the isodual electromagnetic waves requires the existence of antigravity at a pure classical level for their proper identification.

In turn, the experimental confirmation of the gravitational repulsion of light emitted by antimatter would have momentous astrophysical and cosmological implications, since it would permit for the first time theoretical and experimental studies as to whether far away galaxies and quasars are made up of matter or of antimatter.

It is important in this connection to recall that all relativistic quantum field equations admit solutions with positive and negative energies. As it is the case for Dirac's equations, relativistic field equations are generally isoselfdual, thus admitting solutions with both positive and negative energies.

The former are used in numerical predictions, but the negative-energy states are generally discarded because they are believed to be "unphysical".

The isodual theory implies a significant revision of the interpretation of quantum field theory because *the solutions of relativistic equations with positive energy are defined in our spacetime and represent particles, while the joint solutions with* *negative energy are actually defined on the isodual spacetime and represent antiparticles.*

This re-interpretation cannot be presented in this chapter for brevity. In fact, a systematic study of isodual photons requires the formulation of *isodual quantum field theory* that would render prohibitive the length of this chapter.

It is hoped that interested colleagues will indeed work out the proposed isodual quantum field theory, with particular reference to the isodual re-interpretation of advanced and retarded solutions, Green distributions, Feynman diagrams, and all that, because of various implications, such as those in conjugation of trajectories or in the transition from particles to antiparticles.

In closing, the reader should keep in mind that the isodual theory of antimatter resolves all conventional inconsistencies on negative energies as well as against antigravity (see also Section 2.3.15).

2.5 EXPERIMENTAL VERIFICATION OF ANTIGRAVITY

2.5.1 Santilli's Proposed Test of Antigravity for Positrons in Horizontal Flight

By far the most fundamental experiment that can be realized by mankind with current technologies is the measure of the gravitation of truly elementary antiparticles, such as the positron, in the field of Earth.

Irrespective of whether the outcome is positive or negative, the experiment will simply have historical implications for virtually all of physics, from particle physics to cosmology for centuries to come.

If antigravity is experimentally established, the location of the experiment is predicted to become a place of scientific pilgrimage for centuries, due to the far reaching implications, such as the consequential existence of a Causal Time Machine outlined later on in this chapter.

An inspection of the literature soon reveals that the problem of the *gravity of antiparticles in the field of Earth is fundamentally unsettled at this writing, thus requiring an experimental resolution.*

On theoretical grounds, all arguments based on the weak equivalence principle [33] are dismissed as inconclusive by the isodual theory of antimatter, since the latter predicts that bound states of particles and their isoduals experience *attraction* in the gravitational field of Earth.

At any rate, no argument against antigravity based on general relativity can be considered scientifically valid without first the resolution of the catastrophic inconsistencies of gravitation, such as those expressed by the various inconsistency theorems of Section 1.4.

Similarly, all experiments conducted to date on the test of the *gravity of antiparticles not bounded to matter* are equally inconclusive, to the author's best knowledge.[8] A direct measurement of the gravity of positrons was considered in 1967 by Fairbanks and Witteborn [39] via electrons and positrons in a *vertical* vacuum tube.

However, the test could not be conducted because preliminary tests with electrons discouraged the use of positrons due to excessive disturbances caused by stray fields, impossibility of ascertaining the maximal height of the electrons, and other problems.

Neutron interferometric measurements of the *gravity of antiprotons* have been studied by Testera [41], Poggiani [42] and others. However, these experiments are highly sophisticated, thus implying difficulties, such as those for securing antiprotons with the desired *low energies,* magnetic trapping of the antiprotons, highly sensitive interferometric measurements of displacements, and others.

A number of important proposals to text the gravity of antimatter have been submitted to CERN and at other laboratories by T. Goldman, R. J. Hughes, M. M. Nieto, et al. [50–53], although no resolutory measurement has been conducted to date to the author best knowledge, perhaps in view of the excessive ambiguities for an accurate detection of the trajectories of antiparticles under Earth-s gravitational field in existing particle accelerators (see in this respect Figure 2.8).

Additional important references are those studying the connection between antigravity and quantum gravity [54–57], although the latter should be studied by keeping in mind Theorem 1.5.2 on the catastrophic inconsistencies of quantum gravity when realized via nonunitary structures defined on conventional Hilbert spaces and fields.[9]

In view of these unsettled aspects, an experiment that can be *resolutory* with existing technologies, that is, establishing in a final form either the existence of the lack of existence of antigravity, has been proposed by Santilli in Ref. [44] of 1994.

The experiment essentially requires a *horizontal* vacuum tube ranging from 100 meters in length and 0.5 meter in diameter to 10 m in length and 1 m in diameter depending on used energies, with axial collimators at one end and a scintillator at the other end as in Figure 2.7. The proposed test then consists in:

1) Measuring the location in the scintillator of lack of gravitational displacement via a collimated photon beam (since the gravitational displacement on photons at the considered distances is ignorable);

2) Measuring on the same scintillator the downward displacement due to Earth's gravity on an electron beam passing through the same collimators, which

[8]The author would appreciate being kept informed by experimentalist in the field.

[9]The author would like to express his sincere appreciation to T. Goldman for the courtesy of bringing to his attention the important references [50–57] that could not be reviewed here for brevity, but whose study is recommended as a necessary complement of the presentation of this monograph.

downward displacement is visible to the naked eyes for sufficiently small electron energies (for instance, we can have a downward displacement due to gravity of 5 mm, that is visible to the naked eye, for electron kinetic energies of 25 μeV along 100 m horizontal flight, or for electrons with 2 μeV along a 10 m horizontal flight); and

3) Measuring on the same scintillator the displacement due to Earth's gravity on a positron beam passing through the same collimators, which displacement is also visible to the naked eye for positron energies of the order of a few μeV.

If the displacement due to gravity of the positrons is downward, the test would establish the lack of existence of antigravity. On the contrary, the detection of an upward displacement of the positrons would establish the existence of antigravity.

An alternative proposal was submitted by Santilli [20] via the use of the so-called *particle decelerator* in the shape of a doughnut of a diameter of about 10 m and 50 cm in sectional diameter (Figure 2.8). The main idea is that low energy beams of electrons and positrons could be *decelerated* via the use of magnetic fields down to the energy needed to achieve a displacement due to gravity sufficiently larger than the dispersion to be visible to naked eye, at which point the particles are released into a scintillator.

We have stressed throughout this presentation that the only experimental verification of the theoretical prediction of antigravity recommendable at this writing, is that for *truly elementary antiparticles in the gravitational field of matter without any bound to other particles,* such as an isolated beam of positrons under the gravitation field of Earth.

Other tests of antigravity, if conducted before the above tests with positrons and used for general claims on antigravity, can likely lead to ambiguities or a proliferations of unnecessary controversies.

The reasons for this restriction are numerous. Firstly, the study of the gravity of particle-antiparticle systems, such as a bound state of one electron and one positron at large mutual distances according to quantum mechanics (QM),

$$Positronium \,=\, (e^-,\; e^+)_{QM}, \tag{2.5.1}$$

is *strongly discouraged* for a first "test of antigravity", because all theories, including the isodual theory, predict attraction of the positronium in the field of matter. Therefore, under no condition can any possible experimental verification of this prediction be used as a credible claim on the lack of existence of antigravity at large.

Second, the above restriction eliminates the use of muons for a first test of antigravity, because, in view of their instability and decay modes, and as studied in detail in the next chapter, hadronic mechanics (HM) predicts that muons are a bound state of electrons and positrons in conditions of total mutual penetrations

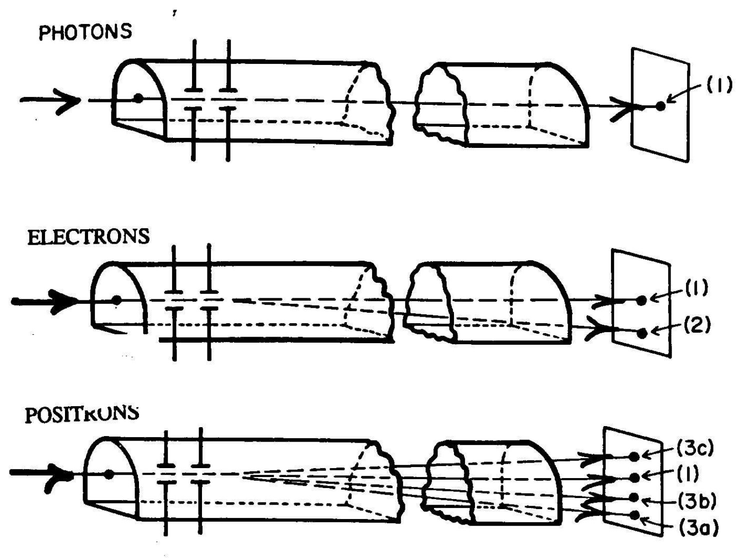

Figure 2.7. A schematic view of the proposal to test the gravity of positrons suggested by Santilli [44] in 1994 via a horizontal vacuum tube with a scintillator at the end in which a collimated beam of photons is used to identify the point in the scintillator of no displacement due to gravity, and collimated beams of very low energy electrons and, separately, positrons are used to measure displacements due to gravity. The latter are indeed visible to the naked eye for sufficiently low kinetic energy of the order of a few μeV. Santilli's proposal [44] was studied by the experimentalist J. P. Mills, jr. [45], as reviewed in the next section.

of their wavepackets at very short mutual distances,

$$\mu^{\pm} = (e^-, \ e^{\pm}, \ e^+)_{HM}, \tag{2.5.2}$$

with consequential highly nonlocal effects structurally beyond any credible treatment by quantum mechanics. Under this structure, *both muons and antimuons are predicted to experience gravitational attraction only* because the possible antigravity of the positron is expected to be less than the gravity of basic electron-positron system.

A similar restriction applies against the use of mesons for first tests of antigravity because they are bound states of particles and antiparticles that, as such, are predicted not to experience antigravity in the field of matter. This is particularly the case for pions. Similarly, a first use of kaons for experiments on antigravity can only result in unnecessary controversies in view of their unsettled structure.

Serious reservation also exist for the first use of antiprotons and antineutrons due to their basically unsettled structure. As stressed earlier, the use of current quark conjecture prevents antiprotons and antineutrons to have any gravity at

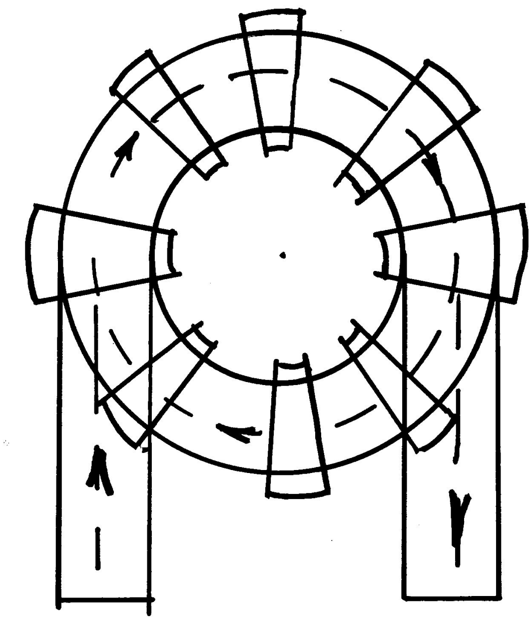

Figure 2.8. A schematic view of the alternative proposal submitted for study by the author [20] at the National High Magnetic Field Laboratory, Tallahassee, Florida, in December 1995. The main idea is to use the established techniques for "particle accelerators" for the construction of a "particle decelerator" that would slow down the initial energy of electron and positron beams down to the amounts needed to produce displacement due to gravity sufficiently bigger than the spread due to stray fields to produce a definite-resolutory answer visible to the naked eye. Suggested dimensions of the "particle decelerator" are 10 m in diameter with a sectional diameter of 0.5 m and two entrances-exits, one used for the entrance-exit of the electron beam and the other for the positron beam. The study conducted by Mills [45] for the horizontal tube indicates that the "particle decelerator" here considered is also feasible and will produce a resolutory answer.

all, let alone antigravity, as rigorously proved by the fact indicated earlier that gravity can only be defined in our physical spacetime while quarks can only be defined in their internal mathematical unitary space, as well as by the lack of credibly defines "quark masses" as inertial eigenvalues of the second order Casimir invariant of the Poincaré group (see the Appendix of Ref. [41]).

Equally equivocal can be at this stage of our knowledge the conduction of first gravitational measurements via the sole use of the antihydrogen atom for intended general results on antigravity, evidently because its nucleus, the antiproton, is believed to be a bound state of quarks for which no gravity at all can be consistently defined. Any study of antigravity under these unsettled structural conditions can only lead to un-necessary controversies, again, if used for general results on antigravity.

It is evident that, *until baryons theories are afflicted by such fundamental problematic aspects, as the inability even to define gravity in a credible way, no gravitational measurement based on antiprotons and antineutrons can be credibly used as conclusive for all of antimatter.*

After the resolution of the gravitational behavior of unbounded positrons in the field of matter, the tests for the gravitational behavior of positronium, muons, muonium, pions, pionium, antiprotons, antineutrons, antihydrogen atom, etc. become essential to acquire an experimental background sufficiently diversified for serious advances on antimatter beyond the level of personal beliefs one way or the other.

The fundamental test of the gravity of positrons here considered was proposed by the author to the following institutions:

1) Stanford Linear Acceleration Center, Stanford, USA, during and following the Seventh Marcel Grossmann Meeting on General Relativity held at Stanford University in July 1994;

2) The Joint Institute for Nuclear Research in Dubna, Russia, during the International Conference on Selected Topics in Nuclear Physics held there in August 1994;

3) The National High Magnetic Field Laboratory in Tallahassee, Florida, during a meeting there in 1996 on magnetic levitation;

4) CERN, Geneva, Switzerland, during a presentation there of hadronic mechanics;;

5) Brookhaven National Laboratories, following the participation at the Sepino meeting on antimatter of 1996 [19];

and to other laboratories as well to universities in various countries.

It is regrettable for mankind that none of these laboratories or universities expressed interest in even considering to date such a fundamental experiment, by preferring to spend much bigger public funds for esoteric experiments manifestly lesser important than that of antigravity.

2.5.2 Santilli's Proposed Tests of Antigravity for Isodual Light

Additionally, in 1997 Santilli [18] predicted that *antimatter emits a new light, the isodual light, that is predicted to be repelled by the gravitational field of matter,* and proposed its experimental verification as the only known (or even conceivable) possibility of ascertaining whether far-away galaxies and quasars are made up of matter or of antimatter.

Measurements as to whether light emitted by the antihydrogen atoms now produced at CERN are attracted or repelled by matter is predictably more delicate than the test of the gravity of the positron, evidently because gravitational displacements for photons in horizontal flight are extremely small, as well know, thus requiring very sensitive interferometric and other measurements.

The experimental detection as to whether far-away galaxies and quasars are made up of matter or of antimatter is predictably more complex and requiring longer periods of time, but with immense scientific implications whatever the outcome.

The test can be done in a variety of ways, one of which consists of measuring the deflection of light originating from far away astrophysical objects when passing near one of our planets. Comparative measurements of a sufficiently large number of galaxies and quasars should permit the detection of possible repulsions, in the event it exists.

Another test has been privately suggested by to the author by an astrophysicist and consists in reinspecting all existing astrophysical data on the deflection of light from far away galaxies and quasars when passing near-by astrophysical bodies.

In the opinion of this astrophysicist, it appears that evidence for the repulsion of light already exists in these data. Such a possible evidence has been ignored so far, and, if found, could not be admitted publicly at the moment, simply because Einstein's gravitation does not allow for any prediction of gravitational repulsion of light.

An understand is that, for these astrophysical measurements to be credible, astrophysicists must conduct the study of a vary large number of galaxies and quasars (of the order of several thousands), and the considered galaxies and quasars must be sufficiently far away to render plausible their possible antimatter structure.

2.5.3 Mills' Studies of Santilli's Proposed Tests of Antigravity

The experimentalist J. P. Mills, jr., [45] conducted a survey of all significant experiments on the gravity of antiparticles in the field of Earth, including indirect tests based on the weak equivalence principle and direct experiments with antiparticles, by concluding that the problem is basically unsettled on theoretical and experimental grounds, thus requiring an experimental resolution.

After considering all existing possible tests, Mills' conclusion is that Santilli's proposed test [44] on the measurement of the gravitational deflection of electrons and positron beams of sufficiently low energy in horizontal flight in a vacuum tube of sufficient length and shielding, is preferable over other possible tests, experimentally feasible with current technology, and providing a resolutory answer as to whether positrons experience gravity or antigravity.

As it is well known, a main technical problem in the realization of Santilli's test is the shielding of the horizontal tube from external electric and magnetic field, and then to have a tube structure in which the internal stray fields have an ignorable impact on the gravitational deflection, or electrons and positrons have such a low energy for which the gravitational deflection is much bigger than possible contributions from internal stray fields, such as the spreading of beams.

The electric field that would cancel the Earth gravitational force on an electron is given by

$$E \ = \ m_e \times g/e = 5.6 \times 10^{-11} \ \mathrm{V/m}. \tag{2.5.3}$$

As it is well known, an effective shielding from stray fields can be obtained via Cu shells. However, our current understanding of the low temperature zero electric field effect in Cu shells does not seem sufficient at this moment to guarantee perfect shielding from stray fields. Mills [45] then suggested the following conservative basic elements for shielding the horizontal tube.

Assuming that the diameter of the tube is d and the shielding enclosure is composed of randomly oriented grains of diameter λ, the statistical variation of the potential on the axis of the tube of a diameter d would then be [45]

$$\Delta V \ = \ \frac{\lambda}{d \times \sqrt{\pi}}. \tag{2.5.4}$$

As expected, *the effect of stray fields at the symmetry axis of the tube is inversely proportional to the tube diameter.* As we shall see, a tube diameter of 0.5 m is acceptable, although one with 1 m diameter would give better results.

Given a work function variation of 0.5 eV, 1 μm grains and d = 30 cm, we would have the following variation of the potential on the axis of the horizontal tube

$$\Delta V \ = 1 \ \mu\mathrm{eV}. \tag{2.5.5}$$

Differences in strain or composition could cause larger variations in stray fields. To obtain significant results without ambiguities for the shielding effect of low temperature Cu shells, Mills [45] suggests *the use of electrons and positrons with kinetic energies significantly bigger than* 1 μeV. As we shall see, this condition is met for tubes with minimal length of 10 m and the diameter of 1 m, although longer tubes would evidently allow bigger accuracies.

The realization of Santilli's horizontal vacuum tube proposed by Mills [45] is the following. As shown in Figure 2.9, the tube would be a long dewar tube, consisting of concentric shells of Al and Mu metals, with Pb and Nb superconducting shells and an inner surface coated with an evaporated Cu film.

There should be two superconducting shells so that they would go superconducting in sequence [Nb (9.25 K), Pb (7.196 K)], evidently for better expulsion of flux. Trim solenoids are also recommended for use within the inner shell and a multitude of connections to the Cu field for trimming electrostatic potentials.

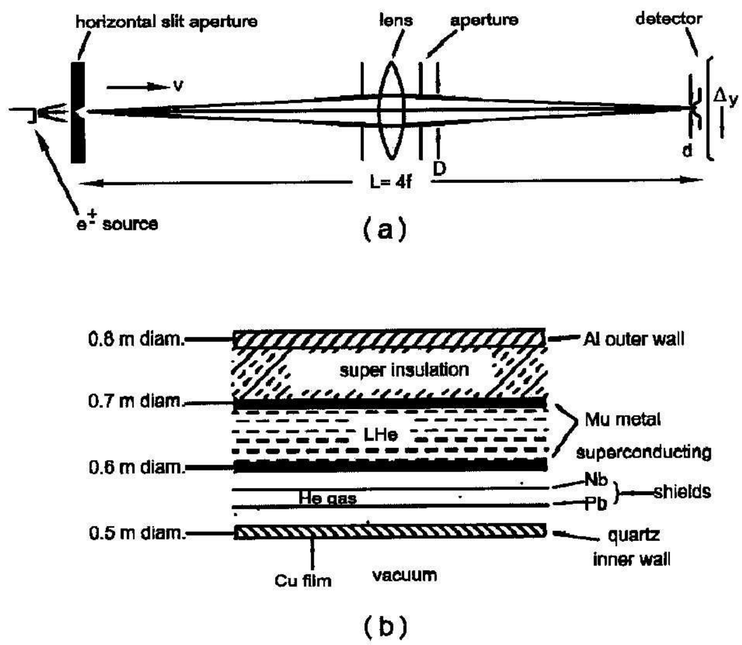

Figure 2.9. A schematic view of the realization suggested by Mills [45] of the horizontal tube proposed by Santilli [44].

As also shown in Figure 2.9, the flight tube should be configured with an electrostatic lens in its center for use of electron and positron beams in both horizontal directions, as well as to focus particles from a source at one end into a gravity deflection sensitive detector at the other end. The de Broglie wavelength of the particles results in the position resolution

$$d \;=\; 2.4 \times \pi \times \alpha_B \times \frac{c \times L}{v \times D}, \tag{2.5.6}$$

where $\alpha = 1/137$ is the fine structure constant, $a_B = 0.529\,\text{Å}$ is the Bohr radius of hydrogen, c is the velocity of light, v is the electron or positron velocity, L is the length of the horizontal path, and D is the diameter of the lens aperture in the center of the flight tube.

The *vertical gravitational deflection* is given by

$$\Delta y \;=\; g \times \frac{L^2}{2 \times v^2}. \tag{2.5.7}$$

Given $L = 100\,\mathrm{m}$, $D = 10\,\mathrm{cm}$, $v/c = 10^{-5}$ (i.e., for 25 μeV particles), we have

$$\Delta y = 5\,\mathrm{mm}. \tag{2.5.8}$$

For 1 meV particles the resolution becomes

$$\Delta y = 125\,\mu\mathrm{m}. \tag{2.5.9}$$

Therefore, one should be able to observe a meaningful deflection using particles with kinetic energies well above the expected untrimmed fluctuation in the potential.

Mills also notes that the lens diameter should be such as to minimize the effect of lens aberration. This requirement, in turn, dictates the minimum inside diameter of the flight tube to be 0.5 m.

The *electron source* should have a cooled field emission tip. A sufficient *positron source* can be provided, for example, by 0.5 ci of ^{22}Na from which we expect (extrapolating to a source five times stronger) $3 \times 10^7\, e^+/\mathrm{s}$ in a one centimeter diameter spot, namely a positron flux sufficient for the test.

Ideal results are obtained when the positrons should be bunched into pulses of 10^4 e^+ at the rate of 10^3 bunches per second. Groups of 10^3 bunches would be collected into macrobunches containing 10^6 e^+ and 20 nsec in duration. The positrons would be removed from the magnetic field and triply brightness enhanced using a final cold Ni field remoderator to give bunches with 10^4 e^+, 10 meV energy spread, an ellipsoidal emission spot 0.1 μm high and 10 μm wide and a 1 radian divergence.

However, stray fields are notoriously weak and decrease rapidly with the distance. Therefore, there is a diameter of the vacuum tube for which stray fields are expected to have value on the axis insufficient to disrupt the test via a spreading of the beams. Consequently, the proposed tests is also expected to be resolutory via the use of very low energy positrons as available, e.g., from radioactive sources.

As a matter of fact, the detection in the scintillator of the same clear gravitational deflection due to gravity by *a few* positrons would be sufficient to achieve a final resolution, provided, of course, that these few events can be systematically reproduced.

After all, the reader should compare the above setting with the fact that new particles are nowadays claimed to be discovered at high energy laboratories via the use of extremely few events out of hundreds of millions of events on record for the same test.

The beam would then be expanded to 100 μm$\times$1 cm cross section and a 1 mrad divergence, still at 10 meV. Using a time dependent retarding potential Mills would then lower the energy spread and mean energy to 100 μeV with a 2 μs pulse width. Even assuming a factor of 1,000 loss of particles due to imperfections in this scheme, Mills' set-up would then have pulses of about 10 positrons that could be launched into the flight tube with high probability of transmissions at energy of 0 to 100 μeV.

The determination of the gravitational force would require many systematic tests. The most significant would be the measurements of the deflection as a function of the time of flight (enhance the velocity v) $\Delta v(e\pm, \pm v)$ for both positrons and elections and for both signs of the velocity relative to the lens on the axis of the tube, $v > 0$ and $v < 0$, the vertical gravitational force on a particle of charge q is

$$F_y \ = \ -m \times g + q \times E_y + q \times v_z \times B_x / c. \tag{2.5.10}$$

The deflection is then given by

$$\Delta y = \int_0^L \int_0^{z'} q \times [E(z'') + v \times B(z'')/c] \\ \times dz'' \times dz'/(m \times v^2) - g \times z^2/2 \times v^2. \tag{2.5.11}$$

In lowest order, Mills neglects the transverse variation in E_y and B_x and writes for the average fields

$$\epsilon \ = \ \frac{1}{L^2} \int_0^L \int_0^{z'} E_y(z'') \times dz'' \times dz', \tag{2.5.12}$$

and

$$\beta \ = \ \frac{1}{L^2} \int_0^L \int_0^{z'} B_x(z'') \times dz'' \times dz'. \tag{2.5.13}$$

Note that these are not simple averages, but the averages of the running averages. They depend on the direction of the velocity. In the approximation that there are not significantly different from simple averages, the average of the four deflection Δy for both positrons and electrons and for both signs of the velocity is independent of ϵ and β and it is given by

$$< \Delta y > = \ (g^+ + g^-) \times \frac{L^2}{v^2}. \tag{2.5.14}$$

where $g^\pm$ refers to the gravitational acceleration of $e^\pm$. Since we also have the velocity dependence of the Δy's, and can manipulate E and B by means of trim adjustments, it will be possible to unravel the gravitational effect from the electromagnetic effect in this experiment.

In summary, the main features proposed by Mills [45] for Santilli's [44] horizontal vacuum tube are that:

1) The tube should be a minimum of 10 m long and 1 m in diameter, although the length of 100 m (as proposed by Santilli [44]) and 0.5 m in diameter is preferable;

2) The tube should contain shields against internal external electric and magnetic fields and internal stray fields. According to Mills [45], this can be accomplished with concentric shells made of Al, double shells of Mu metal, double shells of superconducting Nb and Pb, and a final internal evaporated layer of fine grain of Cu;

3) Use bright pulsed sources of electrons and, separately, positrons, at low temperature by means of phase space manipulation techniques including brightness enhancement;

4) Time of flight and single particle detection should be tested to determine the displacement of a trajectory from the horizontal line as a function of the particle velocity;

5) Comparison of measurements should be done using electrons and positrons traversing the flight tube in both directions.

The *use of electrons and positrons with 25 μeV kinetic energy would yield a vertical displacement of 5 mm at the end of 100 m horizontal flight, namely, a displacement that can be distinguished from displacements caused by stray fields and be visible to the naked eye,* as insisted by Santilli [44].

Mills [45] then concludes by saying that *"... an experiment to measure the gravitational deflection of electrons and positrons in horizontal flight, as suggested by R. M. Santilli, ... is indeed feasible with current technologies.... and should provide a definite resolution to the problem of the passive gravitational field of the positron".*

2.6 SPACETIME LOCOMOTIONS

2.6.1 Introduction

In preceding sections of this monograph we have indicated the far reaching implications of a possible experimental verification of antigravity predicted for antimatter in the field of matter and vice versa, such as a necessary revision of the very theory of antimatter from its classical foundations, a structural revision of any consistent theory of gravitation, a structural revision of any operator formulation of gravitation, and others.

In this section we show that another far reaching implications of the experimental detection of antigravity is the consequential existence of a *Causal Time Machine* [46], that is the capability of moving forward or backward in time without violating the principle of causality, although, as we shall see, this capability is restricted to isoselfdual states (bound states of particles and antiparticles) and *it is not* predicted by the isodual theory to be possible for matter or, separately, for antimatter.

It should be stressed that the Causal Time Machine here considered is a *mathematical model,* rather than an actual machine. Nevertheless, science has always surpassed predictions. Therefore, we are confident that, as it has been the cases for other predictions, one the Causal Time Machine is theoretically predicted, science may indeed permits its actual construction, of course, in due time.

As we shall see, once a Causal Time Machine has been identified, the transition to a causal SpaceTime Machine with the addition of motion in space is direct and immediate.

2.6.2 Causal Time Machine

As clear from the preceding analysis, *antigravity is only possible if antiparticles in general and the gravitational field of antimatter, in particular, evolve backward in time.* A time machine is then a mere consequence.

Causality is readily verified by the isodual theory of antimatter for various reasons. Firstly, *backward time evolution measured with a negative unit of time is as causal as forward time evolution measured with a positive unit of time.* Moreover, *isoselfdual states evolve according to the time of the gravitational field in which they are immersed.* As a result, no violation of causality is conceivably possible for isoselfdual states.

Needless to say, none of these causality conditions are possible for conventional treatments of antimatter.

The reader should be aware that we are referring here to a "Time Machine," that is, to motion forward and backward in time without space displacement (Figure 2.10). The "Space-Time Machine" (that is, including motion in space as well as in time), requires the isodualities as well as isotopies of conventional geometries studied in Chapter 3 and it will be studied in the next section.

The inability to have motion backward in time can be traced back to the very foundations of special relativity, in particular, to the basic time-like interval between two points 1 and 2 in Minkowski space as a condition to verify causality

$$(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 - (t_1 - t_2)^2 \times c^2 < 0. \tag{2.6.1}$$

defined on the field of real numbers $R(n, \times, I),\ I = Diag.(1, 1, 1, 1)$.

The inability to achieve motion backward in time then prevents the achievement of a *closed loop* in the forward light cone, thus including motion in space and time, since said loop would necessarily require motion backward in time.

Consider now an isoselfdual state, such as the positronium or the π° meson (Section 2.3.14). Its characteristics have the sign of the unit of the observer, that is, positive time and energy for matter observers and negative times and negative energies for antimatter observers. Then a closed loop can be achieved as follows [46]:

1) With reference to Figure 2.10, expose first the isoselfdual state to a field of matter, in which case it evolved forward in time from a point at time t_1 to a point at a later time t_2 where the spacetime coordinates verify the time-like invariant (2.6.1) with $t_2 > t_1$;

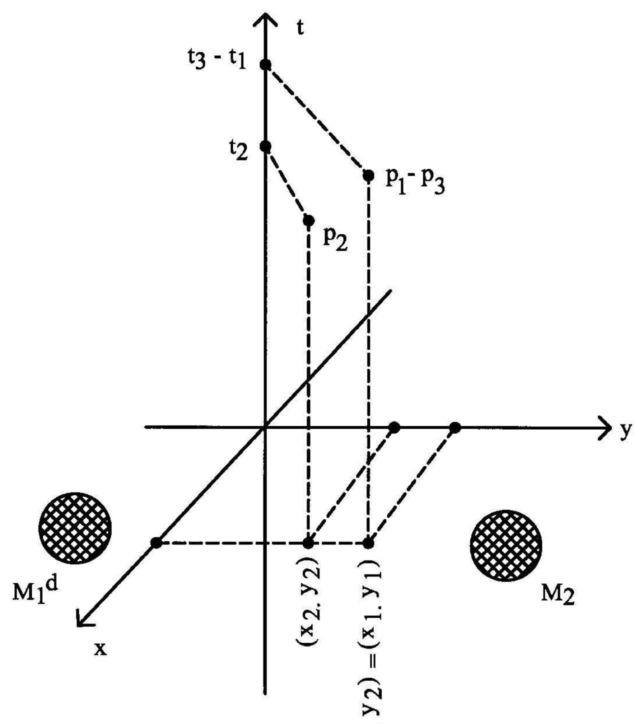

Figure 2.10. A schematic view of the simplest possible version of the "Time Machine" proposed in Ref. [46] via an isoselfdual state such as the positronium or the π° meson that are predicted to move forward (backward) in time when immersed in the gravitational field of matter (antimatter). The Time Machine then follows by a judicious immersion of the same isoselfdual state first in the fields of matter and then in that of antimatter. No causality violation is possible because of the time evolution for isoselfdual states is that of the field in which they are immersed in.

2) Subsequently, expose the same isoselfdual state to a field of antimatter in which case, with the appropriate intensity of the field and the duration of the exposure, the state moves backward in time from time t_2 to the original time t_1, where the spacetime coordinates still verify invariant (2.6.1) with $t_2 < t_1$ although in its isodual form.

We, therefore, have the following:

PREDICTION 2.6.1 [46]: Isoselfdual states can have causal motions forward and backward in time, thus performing causal closed loops in the forward light cone.

Note that the above causal Time Machine implies gravitational *attraction* for both fields of matter and antimatter, owing to the use of an isoselfdual test particle, in which case we only have the reversal of the sign of time and related unit.

Note also that the use of a particle or, separately, of an antiparticle would violate causality.

Numerous time machines exist in the literature. However, none of them appears to verify causality and, as such, they are ignored.

Other time machines are based on exiting our spacetime, entering into a mathematical space (e.g., of complex unitary character), and then returning into our spacetime to complete the loop.

Other attempts have been based on quantum tunnelling effects and other means.

By comparison, the Causal Time Machine proposed in Ref. [46] achieves a closed loop at the classical level without exiting the forward light cone and verifying causality.[10]

2.6.3 Isogeometric Propulsion

All means of locomotion developed by mankind to date, from prehistoric times all the way to current interplanetary missions, have been based on *Newtonian propulsions*, that is, propulsions all based on *Newton's principle of action and reaction.*

As an example, human walking is permitted by the action generated by leg muscles and the reaction caused by the resistance of the feet on the grounds. The same action and reaction is also the origin of *all* other available locomotions, including contemporary automobiles or rockets used for interplanetary missions.

Following the identification of the principle of propulsion, the next central issue is the displacement that is evidently characterized by the *Euclidean distance.* We are here referring to the conventional Euclidean space $E(r,\,\delta,\,R)$ over the reals R with familiar coordinates $r = (x,\,y,\,z) \times I$, metric $\delta = Diag.(1,\,1,\,1)$, units for the three axes $I = I_{3\times 3} = Diag(1\text{ cm},\ 1\text{ cm},\ 1\text{ cm})$ hereon used in their dimensionless form $I = Diag.(1,\,1,\,1)$, and Euclidean distance that we write in the isoinvariant form

$$D^2 = r^2 \times I = (x^2 + y^2 + z^2) \times I \in R. \tag{2.6.2}$$

The *geometric locomotion* can be defined as the *covering of distances via the alteration (also called deformation) of the Euclidean geometry without any use of action and reaction* . The *only* possible realization of such a geometric locomotion that avoid the theorems of catastrophic inconsistencies of Section 1.5, as well as achieves compatibility with our sensory perception (see below), is the *isogeometric*

[10]The indication by colleagues of other versions of the spacetime machine with a proved verification of causality without existing from our spacetime would be appreciated.

locomotion [5b] namely, that permitted by the *Euclid-Santilli isogeometry* and relative *isodistance.*

We are here referring to the Euclid-Santilli isospace (Section 3.2) $\hat{E}(\hat{r}, \hat{\delta}, \hat{R})$ over the isoreals $\hat{R}$ with isocoordinates $\hat{r} = (x, y, z) \times \hat{I}$, metric $\hat{\delta} = \hat{T}_{3\times 3} \times \delta$, isounits for the three isoaxes

$$\hat{I} = \hat{I}_{3\times 3} = Diag(n_1^2\,\mathrm{cm},\ n_2^2\,\mathrm{cm},\ n_3^2\,\mathrm{cm}) = 1/\hat{T}_{3\times 3} > 0 \tag{2.6.3}$$

that will also be used hereon in the dimensionless form

$$\hat{I} = Diag.(n_1^2,\ n_2^2,\ n_3^2), \tag{2.6.4}$$

and *isodistance* that we write in the isoinvariant form[11]

$$\hat{D}^{\hat{2}} = \hat{r}^{\hat{2}} = (x^2/n_1^2 + y^2/n_2^2 + z^2/n_3^2) \times \hat{I} \in \hat{R}, \tag{2.6.5}$$

in which case the deformation of the geometry is called *geometric mutation.*[12]

It is evident that $\hat{D}$ can be bigger equal or smaller than D. Consequently, the isogeometric locomotion occurs when $\hat{D} < D$, as in the example below

$$\hat{I} = Diag.(n_1^2, 1, 1) \ll I = Diag.(1, 1, 1), \quad \hat{T} \gg I, \tag{2.6.6a}$$

$$\hat{D}^{\hat{2}} = (x^2/n_1^2 + y^2 + z^2) \ll D^2 = (x^2 + y^2 + z^2). \tag{2.6.6b}$$

The understanding of the above locomotion requires a knowledge of the *isobox* of Section 3.2. Consider such an isobox and assume that it is equipped with isogeometric locomotion. In this case, there is no displacement at all that can be detected by the internal observer. However, the external observer detects a displacement of the isobox the amount $x^2 - x^2/n_1^2$.

This type of locomotion is new because it is causal, invariant and occurs without any use of the principle of action and reaction and it is geometric because it occurs via the sole local mutation of the geometry.

The extension to the *Causal Spacetime Machine,* or *spacetime isogeometric locomotion* is intriguing, and can be formulated via the *Minkowski-Santilli isospace* of Section 3.2 with four-isodistance

$$\hat{D}^{\hat{2}} = (x^2/n_1^2 + y^2/n_2^2 + z^2/n_3^2 - c^2 \times t^2/n_4^2) \times \hat{I} \in \hat{R}, \tag{2.6.7}$$

---

[11] By "isoinvariance" we means invariance under conventional space or spacetime symmetries plus the isotopic invariance.

[12] According to the contemporary terminology, "deformations" are alterations of the original structure although referred to the original field. As such they are afflicted by the catastrophic inconsistencies of Section 1.5. The term "mutation", first introduced by Santilli in Ref. [49] of 1967, is today referred to an alteration of the original structure under the condition of preserving the original axioms, thus requiring the formulation on isospaces over isofields that avoid said theorems of catastrophic inconsistency.

Figure 2.11. An artistic rendering of the "Space Time Machine", namely, the "mathematical" prediction of traveling in space and time permitted by the isodual theory of antimatter. The main assumption is that the aether (empty space) is a universal medium characterized by a very high density of positive and negative energies that can coexist because existing in distinct, mutually isodual spacetimes. Virtually arbitrary trajectories and speeds for isoselfdual states (only) are then predicted from the capability of extracting from the aether very high densities of positive and negative energies in the needed sequence. Discontinuous trajectories do not violate the law of inertia, speeds much bigger than the speed of light in vacuum, and similarly apparently anomalous events, do not violate special relativity because the locomotion is caused by the change of the local geometry and not by conventional Newtonian motions.

where $n_4 > 0$.

The main implications in this case is the emergence of the additional *time mutation* as expected to occur jointly with any *space mutation.* In turn, this implies that the *isotime* $\hat{t} = t/n_4$ (that is, the internal time) can be bigger equal or smaller than the time t (that of the external observer).

More specifically, from the preservation of the original trace of the metric, *isorelativity predicts that the mutations of space and time are inversely promotional to each others.* Therefore, jointly with the motion ahead in space there is a motion backward in time and vice versa.

Consequently, the external observer sees the object moving with his naked eye, and believes that the object evolves in his own time, while in reality the object could evolve far in the past. Alternatively, we can say that the inspection of an astrophysical object with a telescope, by no means, implies that said object evolves with our own time because it could evolve with a time dramatically different than that after adjustments due to the travel time of light because, again, light cannot carry any information on the actual time of its source.

To further clarify this important point, *light cannot possibly carry information on the time of its source because light propagates at the speed c at which there is no time evolution.*

As a concrete example, one of the consequences of interior gravitational problems treated via Santilli's isorelativity (see Section 3.5) is that *the time of interior gravitational problems, $\hat{t} = t/n_4$, depends on the interior density n_4^2, rather than the inertial mass, thus varying for astrophysical bodies with different densities.*

This implies that if two identical watches are originally synchronized with each other on Earth, and then placed in the interior gravitational field of astrophysical bodies with different densities, they will no longer be synchronized, thus evolving with different times, even though light may continue to provide the information needed for their intercommunication.

In particular, *the time evolution of astrophysical bodies slows down with the increase of the density*,

$$\hat{t}_1 < \hat{t}_2, \quad n_{41}^2 > n_{42}^2. \tag{2.6.8}$$

It should also be noted that the above effect has no connection with similar Riemannian predictions because it is structurally dependent on the *change of the units,* rather than geometric features.

A prediction of isospecial relativity is that the bigger the density, the slower the time evolution. Thus, a watch in the interior of Jupiter is predicted to move *slower* than its twin on Earth under the assumption that the density of Jupiter (being a gaseous body) is significantly smaller than that of Earth (that can be assumed to be solid for these aspects).

As stressed in Section 2.6.1, *the above spacetime machine is a purely mathematical model*. To render it a reality, there is the need to identify the *isogeometric propulsion*, namely a source for the geometric mutations of type (2.6.5).

Needless to say, the above problem cannot be quantitatively treated on grounds of available scientific knowledge. However, to stimulate the imagination of readers with young minds of any age, a speculation on the possible mechanism of propulsion should be here voiced.

The only source of geometric mutation conceivable today is the availability of very large energies concentrated in very small regions of space, such as energies of the order of 10^{30} ergs/cm^3. Under these conditions, isorelativity does indeed predict isogeometric locomotion because these values of energy density generate very large values of isounits $\hat{I}$, with very small values of the isotopic element $\hat{T}$, resulting in isogeometric locomotions precisely of type (2.6.5).

The only possible source of energy densities of such extreme value is empty space. In fact, according to current views, space is a superposition of positive and negative energies in equal amounts each having extreme densities precisely of the magnitude needed for isogeometric locomotion.

The speculation that should not be omitted in this section is therefore that, one day in the future, the advancement of science will indeed allow to extract from space at will all needed amounts of both positive and negative energy densities.

In the event such an extraction becomes possible in a directional way, a spaceship would be able to perform all desired types of trajectories, including trajectories with sharp discontinuities (instantaneous 90 degrees turns), instantaneous accelerations, and the like without any violation of the law of inertia because, as indicated earlier, the spaceship perceives no motion at all. It is the geometry in its surroundings that has changed.

Moreover, such a spaceship would be able to cover interstellar distances in a few of our minutes, although arriving at destination way back in the time evolution of the reached system.

Science has always surpassed science fiction and always will, because there is no limit to the advancement of scientific knowledge. On this ground it is, therefore, easy to predict that, yes, one day mankind will indeed be able to reach far away stars in minutes.

It is only hoped that, when that giant step for mankind is achieved, the theory that first achieved its quantitative and invariant prediction, Santilli isorelativity, will be remembered.

References

[1] R. M. Santilli, Hadronic J. **8**, 25 and 36 (1985).

[2] R. M. Santilli, Algebras, Groups and Geometries **10**, 273 (1993).

[3] J. V. Kadeisvili, Algebras, Groups and Geometries **9**, 283 and 319 (1992).

[4] R. M. Santilli, Rendiconti Circolo Matematico di Palermo, Supplemento **42**, 7 (1996).

[5] R. M. Santilli, *Elements of Hadronic Mechanics*, Vols. I [5a] and II [5b], Ukrainian Academy of Sciences, Kiev, Second Edition (1995).

[6] R. M. Santilli, Comm. Theor. Phys. **3**, 153 (1993).

[7] R. M. Santilli, J. Moscow Phys. Soc. **3**, 255 (1993).

[8] R. M. Santilli, Intern. J. Modern Phys. D **7**, 351 (1998).

[9] R. M. Santilli, Intern. J. Modern Phys. A **14**, 2205 (1999).

[10] D. V. Ahluwalia, M. B. Johnson and T. Goldman, Phys. Letters B **316**, 102 (1993).

[11] J. Dunning-Davies, Foundations of Physics Letters, **12**, 593 (1999).

[12] E. Corinaldesi and F. Strocchi, *Relativistic Wave Mechanics,* North Holland, Amsterdam (1963).

[13] E. Majorana, Nuovo Cimento **14**, 171 (1937).

[14] J. A. McLennan, jr., Phys. Rev. **106**, 821 (1957).

[15] K. M. Case, Phys. Rev. **107**, 307 (1957).

[16] D. V. Ahluwalia, Intern. J. Modern Phys. A **11**, 1855 (1996).

[17] V. V. Dvoeglazov, Hadronic J. **20**, 435 (1997).

[18] R. M. Santilli, Hyperfine Interactions, **109**, 63 (1997).

[19] M. Holzscheiter, Editor, *Proceedings of the International Workshop on Antimatter Gravity*, Sepino, Molise, Italy, May 1996, Hyperfine Interactions, Vol. **109** (1997).

[20] R. M. Santilli, contributed paper to *New Frontiers in Hadronic Mechanics*, T. L. Gill, Editor, Hadronic Press (1996), pp. 343–416.

[21] R. M. Santilli, contributed paper to *Proceedings of the International Workshop on Modern Modified Theories of Gravitation and Cosmology,* E. I. Guendelman, Editor, Hadronic Press, pages 113-169 (1998).

[22] R. M. Santilli, Journal of Dynamical Systems and Geometric Theories, **1**, 121 (2003).

[23] J. V. Kadeisvili, Algebras, Groups and Geometries **9**, 283 and 319 (1992).

[24] J. V. Kadeisvili, Math. Methods in Appl. Sci. **19**, 1349 (1996).

[25] J. V. Kadeisvili, Acta Appl. Math. **50**, 131 (1998).

[26] A. K. Aringazin, A. Jannussis, D. F. Lopez, M. Nishioka and B. Veljanosky, *Santilli's Lie-Isotopic Generalization of Galilei's Relativities,* Kostarakis Publisher, Athens, Greece (1980) [10a].

[27] J. V. Kadeisvili, *Santilli's Isotopies of Contemporary Algebras, Geometries and Relativities*, Second Edition, Ukrainian Academy of Sciences, Kiev (1997).

[28] D. S. Sourlas and G. T. Tsagas, *Mathematical Foundations of the Lie-Santilli Theory*, Ukrainian Academy of Sciences, Kiev (1993).

[29] J. Lôhmus, E. Paal and L. Sorgsepp, *Nonassociative Algebras in Physics*, Hadronic Press, Palm Harbor, FL, (1994).

[30] R. M. Falcon Ganfornina and J. Nunez Valdes, *Fondamentos de la Isoteoria de Lie-Santilli,* (in Spanish) International Academic Press, America-Europe-Asia, (2001), also available in the pdf file http://www.i-b-r.org/docs/spanish.pdf.

[31] Chun-Xuan Jiang, *Foundations of Santilli's Isonumber Theory, with Applications to New Cryptograms, Fermat's Theorem and Goldbach's Conjecture*, International Academic Press, America-Europe-Asia (2002), also available in the pdf file http://www.i-b-r.org/docs/jiang.pdf.

[32] W. Pauli, *Theory of Relativity*, Pergamon Press, London (1958).

[33] C. W. Misner, K. S. Thorne and A. Wheeler, *Gravitation*, Freeman, San Francisco (1970).

[34] R. M. Santilli, *Isodual Theory of Antimatter with Applications to Antigravity, Spacetime Machine and Cosmology,* Springer (2006).

[35] D. H. Childress, *The Antigravity Handbook,* Network Unlimited Press, Stell, Illinois (1983).

[36] B. C. Ebershaw, *Antigravity Propulsion Devices,* D&B Associates, New York (1980).

[37] J. Sherk, Phys. Letters B **88,** 265 (1979).

[38] J. Sherk, contributed paper in *Supergravity,* P. van Neuwenhulzen and D. Z. Freedoman, Editors, North Holland, Amsterdam (1979).

[39] W. M. Fairbank and F. C. Witteborn, Phys. Rev. Lett. **19**, 1049 (1967).

[40] R. E. Brown et al., Nucl. Instr. Methods, Phys. Res. B **26**, 480 (1991).

[41] G. Testera, Hyperfine Interactions **109,** 333 (1997).

[42] R. Poggiani, Hyperfine Interactions **109,** 367 (1997).

[43] M. M. Nieto and T. Goldman, Phys. Reports **205,** 221 (1991), erratum **216**, 343 (1992).

[44] R. M. Santilli, Hadronic J. **17**, 257 (1994).

[45] J. P. Mills, jr., Hadronic J. **19**, 1 (1996).

[46] R. M. Santilli, Hadronic J. **17**, 285 (1994).

[47] D. Lovelock and H. Rund, *Tensors, Differential Forms and Variational Principles*, Wiley, New York (1975).

[48] R. M. Santilli, Ann. Phys. **83**, 108 (1974).

[49] R. M. Santilli, Nuovo Cimento **51**, 570 (1967).

[50] T. Goldman and M. M. Nieto, Phys. Lett. **112B**, 437 (1982).

[51] T. Goldman, M. V. Hynes and M. M. Nieto, Gen. Rev. Grav. **18** 67 (1986).

[52] N. Beverini et al., "A Measurement of the Gravitational Acceleration of the Antiproton," LA-UR-86-260, experimental proposal submitted to CERN.

[53] T. Goldman, R. J. Hughes and M. M. Nieto, Phys. Rev. D **36**, 1254 (1987).

[54] T. Goldman, R. J. Hughes and M. M. Nieto, Phys. Lett. **171B**, 217 (1986).

[55] T. Goldman, R. J. Hughes and M. M. Nieto, in *Proceedings of the First Workshop on Antimatter Physics at Low Energy*, ed. by B. E. Bonner, and L. S. Pinsky, Fermilab (April 1986), p. 185.

[56] T. Goldman, R. J. Hughes and M. M. Nieto, in *Proceedings of Second Conf. on the Intersections Between Particle and Nuclear Physics*, (Lake Louise, May 1986), AIP Conf. Proc. 150, ed. by D. Geesaman, p. 434.

[57] T. Goldman, R. J. Hughes and M. M. Nieto, in *Proceedings of the GR11 Conference*, Stockholm July 1986.

Postscript

In the history of science some basic advances in physics have been preceded by basic advances in mathematics, such as Newtons invention of calculus and general relativity relying on Riemannian geometry. In the case of quantum mechanics the scientific revolution presupposed the earlier invention of complex numbers. With new numbers and more powerful mathematics to its disposition, physics could be lifted to explain broader and more complex domains of physical reality.

The recent and ongoing revolution of physics, initiated by Prof. Ruggero Maria Santilli, lifting the discipline from quantum mechanics to hadronic mechanics, is consistent with this pattern, but in a more far-reaching and radical way than earlier liftings of physics made possible from extensions of mathematics.

Santilli realized at an early stage that basic advances in physics required invention of new classes of numbers and more adequate and powerful mathematics stemming from this. His efforts to develop such expansions of mathematics started already in 1967, and this enterprise went on for four decades. Its basic novelties, architecture and fruits are presented in the present volume. During this period a few dozen professional mathematicians world wide have made more or less significant contributions to fill in the new Santilli fields of mathematics, but the honor of discovering these vast new continents and work out their basic topology is Santillis and his alone. These new fields initiated by Santilli made possible realization of so-called Lie-admissible physics. For this achievement Santilli in 1990 received the honor from Estonia Academy of Science of being appointed as mathematician number seven after world war two considered a landmark in the history of algebra.

With regard to Sophus Lie it may be of some interest to note that the Norwegian examiners of his groundbreaking doctoral thesis in 1871 were not able to grasp his work, due to its high degree of novelty and unfamiliarity. However, due to Lie already being highly esteemed among influential contemporary mathematicians at the continent, it was not an option to dismiss his thesis. As in other disciplines, highly acknowledged after Thomas Kuhns publication of The structure of scientific revolutions in 1962, sufficiently novel mathematics implies some paradigmatic challenge. Therefore, it is not strange that some mathematicians and physicists have experienced difficulties taking the paradigmatic leap necessary to grasp the basics of hadronic mathematics or to acknowledge its far-reaching implications. Such a challenge is more demanding when scientific novelty implies a reconfiguration of conventional basic notions in the discipline. This is, as Kuhn noted, typically easier for younger and more emergent scientific minds.

Until Santilli the number 1 was silently taken for granted as the primary unit of mathematics. However, as noted by mathematical physicist Peter Rowlands at University of Liverpool, the number 1 is already loaded with assumptions, that can be worked out from a lifted and broader mathematical framework. A partial and rough analogy might be linguistics where it is obvious that a universal science of language must be worked out from a level of abstraction that is higher than having to assume the word for mother to be the first word.

Santilli detrivialized the choice of the unit, and invented isomathematics where the crux was the lifting of the conventional multiplicative unit (i.e. conservation of its topological properties) to a matrix isounit with additional arbitrary functional dependence on other needed variables. Then the conventional unit could be described as a projection and deformation from the isounit by the link provided by the so-called isotopic element inverse of the isounit. This represented the creation of a new branch of mathematics sophisticated and flexible enough to treat systems entailing sub-systems with different units, i.e. more complex systems of nature.

Isomathematics proved necessary for the lifting of quantum mechanics to hadronic mechanics. With this new mathematics it was possible to describe extended particles and abandon the point particle simplification of quantum mechanics. This proved highly successful in explaining the strong force by leaving behind the non-linear complexities involved in quantum mechanics struggle to describe the relation between the three baryon quarks in the proton. Isomathematics also provided the mathematical means to explain the neutron as a bound state of a proton and an electron as suggested by Rutherford. By means of isomathematics Santilli was also able to discover the fifth force of nature (in cooperation with Professor Animalu), the contact force inducing total overlap between the wave packets of the two touching electrons constituting the isoelectron. This was the key to understanding hadronic superconductivity which also can take place in fluids and gases, i.e. at really high temperatures. These advances from hadronic mechanics led to a corresponding lifting of quantum chemistry to hadronic chemistry and the discovery of the new chemical species of magnecules with non-valence bounds. Powerful industrial-ecological technology exploiting these theoretical insights was invented by Santilli himself from 1998 on.

Thus, the development of hadronic mathematics by Santilli was not only motivated by making advances in mathematics per se, but also of its potential to facilitate basic advances in physics and beyond. These advances have been shown to be highly successful already. Without the preceding advances in mathematics, the new hadronic technology would not have been around. The mere existence of this technology is sufficient to demonstrate the significance of hadronic mathematics. It is interesting to note that the directing of creative mathematics into this path was initiated by a mathematical physicist, not by a pure mathematician. In general this may indicate the particular potential for mathematical advances by relating the mathematics to unsolved basic problems in other disciplines, as well as to real life challenges.

In the history of mathematics it is not so easy to find parallels to the achievements made by Santilli, due to hadronic mathematics representing a radical and general lifting, relegating the previous mathematics to a subclass of isomathematics, in some analogy to taking the step from the Earth to the solar system. However, the universe also includes other solar systems as well as galaxies.

In addition to isonumbers Santilli invented the new and broader class of genonumbers with the possibility of asymmetric genounits for forward vs. backward genofields, and designed to describe and explain irreversibility, characteristic for more complex systems of nature. Quantum mechanical approaches to biological systems never achieved appreciable success, mainly due to being restricted by a basic symmetry and hence reversibility in connected mathematical axioms. It represented an outstanding achievement of theoretical biology when Chris Illert in the mid-1990s was able to find the universal algorithm for growth of sea shells by applying hadronic geometry. Such an achievement was argued not to be possible for more restricted hyperdimensional geometries as for example the Riemannian. This specialist study in conchology was the first striking illustration of the potency as well as necessity of iso- and genomathematics to explain irreversible systems in biology.

Following the lifting from isomathematics to genomathematics, Santilli also established one further lifting, by inventing the new and broader class of hyperstructural numbers or Santilli hypernumbers. Such hypernumbers are multivalued and suitable to describe and explain even more complex systems of nature than possible with genonumbers. Due to its irreversible multivalued structure hypermathematics seems highly promising for specialist advances in fields such as genetics, memetics and communication theory. By the lifting to hypermathematics hadronic mathematics as a whole may be interpreted as a remarkable step forward in the history of mathematics, in the sense of providing the essential and sufficiently advanced and adequate tools for mathematics to expand into disciplines such as anthropology, psychology and sociology. In this way it is possible to imagine some significant bridging between the two cultures of science: the hard and the soft disciplines, and thus amplifying a tendency already represented to some extent by complexity science.

The conventional view of natural scientists has been to regard mathematics as a convenient bag of tools to be applied for their specific purposes. Considering the architecture of hadronic mathematics, this appears more as only half of the truth or one side of the coin. Besides representing powerful new tools to study nature, hadronic mathematics also manifests with a more intimate and inherent connection to physics (and other disciplines), as well as to Nature itself. In this regard hadronic geometry may be of special interest as an illustration:

Isogeometry provided the new notions of a supra-Euclidean isospace as well as its anti-isomorphic isodual space, and the mathematics to describe projections and deformations of geometrical relations from isospace and its isodual into Euclidean space. However, these appear as more than mere mathematical constructs. Illert showed that the universal growth pattern of sea shells could be found only by looking for it as a trajectory in a hidden isospace, a trajectory which is projected into Euclidean space and thereby manifest as the deformed growth patterns humans observe by their senses. Further, the growth pattern of a certain class of sea shells (with bifurcations) could only be understood from the addition and recognition of four new, non-trivial time categories (predicted to be discovered by hadronic mechanics) which manifest as information jumps back and forth in Euclidean space. With regard to sea shell growth, one of this non-trivial time flows could only be explained as a projection from isodual spacetime. This result was consistent with the physics of hadronic mechanics, analyzing masses at both operator and classical level from considering matter and anti-matter (as well as positive and negative energy) to exist on an equal footing in our universe as a whole and hence with total mass (as well as energy and time) cancelling out as zero for the total universe. To establish a basic physical comprehension of Euclidean space constituted as a balanced combination of matter and antimatter, it was required to develop new mathematics with isonumbers and isodual numbers basically mirroring each other. Later, corresponding anti-isomorphies were achieved for genonumbers and hypernumbers with their respective isoduals.

Thus, there is a striking and intimate correspondence between the isodual architecture of hadronic mathematics and the isodual architecture of hadronic mechanics (as well as of hadronic chemistry and hadronic biology). Considering this, one might claim that the Santilli inventions of new number fields in mathematics represent more than mere inventions or constructs, namely discoveries and reconstructions of an ontological architecture being for real also outside the formal landscapes created by the imagination of mathematics and logic. This opens new horizons for treating profound issues in cosmology and ontology.

One might say that with the rise of hadronic mathematics the line between mathematics and other disciplines has turned more blurred or dotted. In some respect this represents a revisit to the Pythagorean and Platonic foundations of mathematics in the birth of western civilization. Hadronic mathematics has provided much new food for thought and further explorations for philosophers of science and mathematics.

If our civilization is to survive despite its current problems, it seems reasonable to expect Santilli to be honored in future history books not only as a giant in the general history of science, but also in the specific history of mathematics. Hadronic mathematics provided the necessary fuel for rising scientific revolutions in other hadronic sciences. This is mathematics that matters for the future of our world, and hopefully Santillis extraordinary contributions to mathematics will catch fire among talented and ambitious young mathematicians for further advances to be made. The present mellowed volume ought to serve as an excellent appetizer in this regard.

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